Brackets (7-9)

Contents

Introduction

We have already seen that brackets are needed in certain expressions:

I think of a number, add 8 and multiply by 4 ... $4(x+8)$

It is possible to expand brackets so that the expression no longer has brackets.

1 Single brackets (7-9)

Consider the sum $3\times(2+4)$. We can see the answer is $3\times 6=18$. This can be worked out in another way:

$$\begin{aligned}3\times(2+4) &= 3\times 2+3\times 4\\ &= 6+12\\ &= 18\end{aligned}$$

We can see that each number in the bracket must be multiplied by the number outside. We can extend this into algebra:

$$\begin{aligned}3(a+4) &= 3\times a+3\times 4\\ &= 3a+12\end{aligned}$$

$$6(m-2) = 6m-12$$

$$x(x+5) = x^2+5x$$

If an expression contains more than one set of bracket, these can be expanded separately and then the expression can be simplified by collecting the like terms:

$$\begin{aligned}5(x+9)+9(x-2) &= 5x+45+9x-18\\ &= 5x+9x+45-18\\ &= 14x+27\end{aligned}$$

2 Single brackets with a negative outside (8-9)

Be very careful when there is a negative term outside a bracket. Consider first this example with numbers:

$$\begin{aligned} -3 \times (9-5) &= -3 \times 4 \\ &= -12 \end{aligned}$$

Or

$$\begin{aligned} -3x(9-5) &= (-3) \times 9 - (-3) \times 5 \\ &= -27(-15) \\ &= -27 + 15 \\ &= -12 \end{aligned}$$

We can see that the 9 has become -27 after expanding and the -5 has become $+15$. That is, positive terms become negative and negative terms become positive when there is a negative outside the brackets.
We can extend this to algebra. We will use $\oplus$ to stand for positive and $\ominus$ for negative in this example:

$$\begin{aligned} -7(x-2) &= -7(\overbrace{x}^{\oplus}\overbrace{-2}^{\ominus}) \\ &= \overbrace{-7x}^{\ominus}\overbrace{+14}^{\oplus} \end{aligned}$$

Again notice how negative terms have become positive and vice versa.

$$-6(y+8) = -6y - 48$$

$$-5(m-7) = -5m + 35$$

$$\begin{aligned} 5(a+9) - 3(2-a) &= 5a + 45 - 6 + 3a \\ &= 5a + 3a + 45 - 6 \\ &= 8a + 39 \end{aligned}$$

3 Double brackets (8-9)

We can have a set of double brackets in algebra:

$$(x+3)(x+2)$$

Do not confuse this with two sets of single brackets:

$$(x+3) + (x+2)$$

Double brackets must have a multiplication between them.
Consider 11×12; we know the answer to this is 132: lets see how we can get 132 using double brackets:

$$\begin{aligned} 11 \times 12 &= (10+1) \times (10+2) \\ &= 10 \times 10 + 10 \times 2 + 1 \times 10 + 1 \times 2 \\ &= 100 + 20 + 10 + 2 \\ &= 132 \end{aligned}$$

We can see that every term in one bracket must be multiplied by every term in the other, giving four pairs altogether. A good way to remember this is **F.O.I.L.**

F	first from each bracket	(here, 10×10)
O	outer most terms	(here 10×2)
I	inner most terms	(here 1×10)
L	last from each bracket	(here 1×2)

Using **F.O.I.L.** we can expand double brackets involving algebraic terms:

$$\begin{aligned} (p+3)(p+2) &= \overbrace{(p \times p)}^{F} + \overbrace{(p \times 2)}^{O} + \overbrace{(3 \times p)}^{I} + \overbrace{(3 \times 2)}^{L} \\ &= p^2 + 2p + 3p + 6 \\ &= p^2 + 5p + 6 \end{aligned}$$

Here are more examples:

$$\begin{aligned} (m+4)(m-2) &= m^2 - 2m + 4m - 8 \\ &= m^2 + 2m - 8 \end{aligned}$$

$$\begin{aligned} (k-3)(k-4) &= k^2 - 4k - 3k + 12 \\ &= k^2 - 7k + 12 \end{aligned}$$

A common mistake — we must multiply every pair, not add them:

$(x+5)(x+2) = x^2 + 2x + 5x + 7$ *WRONG*

$(x+5)(x+2) = x^2 + 2x + 5x + 10$ *CORRECT*

Some expressions do not appear to be double brackets, but they are double brackets in disguise:

$(x+4)^2$ ... you may want to write $x^2 + 16$
But $(x+4)^2$ means $(x+4)(x+4)$.

$$\begin{aligned} (x+4)^2 &= (x+4)(x+4) \\ &= x^2 + 4x + 4x + 16 \\ &= x^2 + 8x + 16 \end{aligned}$$

Challenge. Can you put all of your brackets skills together and try to simplify

$$(y-3)^2 - 7(y-9)?$$

$$\begin{aligned}(y-3)^2 - 7(y-9) &= (y-3)(y-3) - 7(y-9)\\ &= y^2 - 3y - 3y + 9 - 7y + 63\\ &= y^2 - 3y - 3y - 7y + 9 + 63\\ &= y^2 - 13y + 72\end{aligned}$$

4 Equations involving double brackets

Follow this example:
Solve $(x+5)^2 = (x+6)(x-4)$.

$$\begin{aligned}(x+5)^2 &= (x+6)(x-4)\\ (x+5)(x+5) &= (x+6)(x-4)\\ x^2 + 5x + 5x + 25 &= x^2 - 4x + 6x - 24\\ x^2 + 10x + 25 &= x^2 + 2x - 24\\ 10x + 25 &= 2x - 24 \text{ (}x^2\text{ cancels from each side)}\\ 8x + 25 &= -24\\ 8x &= -49\\ x &= -\frac{49}{8}\\ x &= -6\frac{1}{8}\end{aligned}$$

N.B. Do not confuse $(x+5)^2 + (x+6)(x-4)$ and $(x+5)^2 = (x+6)(x-4)$. The first is an expression which can be expanded and simplified. The second is an equation so the x^2 term will cancel since it can be subtracted from both sides of the equation.