Formulæ(7-9)

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1 Writing and Using formulæ(7)

A formula is recognised statement that expresses the relationship between certain variables. E.g.

$$\begin{aligned}\text{Area of a triangle} &= \frac{1}{2}\times\text{base}\times\text{perpendicular height}\\ A &= \frac{1}{2}bh\end{aligned}$$

In the above example, A is called the *subject* of the formula since it names the formula. If we know the value of each variable in the formula, we can work out the value of the subject.

Example. A hotel charges guests according to the formula:

$$C = 50 + 30n,$$

where C is the cost in pounds and n is no. of nights stay. How much would it cost for a 4 night stay in this hotel?

$$\begin{aligned}C &= 50+30n\\ C &= 50+30\times 4 && \textit{Remember BODMAS}\\ C &= 50+120\\ C &= £170\end{aligned}$$

If we know the value of the subject, it may be possible to work out the value of a different variable. E.g. Using the above formula, how many nights did I stay at the hotel if my bill came to $£260$?

$$\begin{aligned}C &= 50+30n\\ 260 &= 50+30n && \textit{Read like a one-sided equation}\\ 210 &= 30n\\ n &= 7\end{aligned}$$

It is possible to write formulae of your own using given information. Dont forget that a formula must have an equals sign and that it is named by a capital letter.

Example. Write a formula for the perimeter of an equilateral triangle with side x.

$$\begin{aligned} \text{Perimeter} &= \text{Total of all sides} \\ P &= x + x + x \\ P &= 3x \end{aligned}$$

2 Changing the subject of a formula (8 & 9)

Consider the formula for the area of a circle:

$$A = \pi r^2.$$

At the moment, A is the subject of the formula. It is easy to work out the area of the circle, A, if we know its radius, r. However, what if we wanted to work out r instead? In this case it may be best to rearrange the formula first to make r the subject.

2.1 Formulae that simply require reading

Most formulae can be "read" (like one-sided equations) and the layers undone in reverse order (the last layer that is added is the first one to be undone – just like you put your coat on last each morning but it is the first layer to be taken off when you get home). Follow these examples:

Example. Make x the subject of the formula $mx + k = q$:

1. Read the algebra from x: *I think of a number, multiply it by m and add k*
2. Peel off the layers, doing the same thing to both sides:

$$\begin{aligned} mx + k &= q && \textit{Subtract } k \textit{ from both sides} \\ mx &= q - k && \textit{Divide both sides by } m \\ x &= \tfrac{q-k}{m} \end{aligned}$$

Example. Make y the subject of the formula $\frac{y}{t} + l = q$.

1. Read the algebra from y: *I think of a number, divide it by t and add l*
2. Peel off the layers, doing the same thing to both sides:

$$\begin{aligned} \tfrac{y}{t} &= q - l && \textit{Subtract } l \textit{ from both sides} \\ y &= t(q - l) && \textit{Multiply both sides by } t \end{aligned}$$

We know that roots and powers undo each other:

Example. Make t the subject of the formula $\sqrt{t} - k = m$.

1. Read the algebra from t: *I think of a number, square root it and subtract* k
2. Peel off the layers, doing the same thing to both sides:

$$\begin{aligned} \sqrt{t} &= m + k & \textit{Add } k \textit{ to both sides} \\ t &= (m+k)^2 & \textit{Square both sides} \end{aligned}$$

Example. Make w the subject of the formula $mw^3 = t$.

1. Read the algebra from w: *I think of a number, cube it and multiply it by* m
2. Peel off the layers, doing the same thing to both sides:

$$\begin{aligned} w^3 &= \tfrac{t}{m} & \textit{Divide both sides by } m \\ w &= \sqrt[3]{\tfrac{t}{m}} & \textit{Cube root both sides} \end{aligned}$$

2.2 A few little things to look out for

If the formula to be arranged involves a fraction, it would look quite "messy" to deal with it in the simplest way. E.g.

$$\begin{aligned} \frac{1}{4}m &= t \\ m &= \frac{t}{\frac{1}{4}} \end{aligned}$$

We must remember that dividing by $\frac{1}{4}$ is the same as multiplying by 4 (to divide by a fraction, multiply by its reciprocal). Hence, this rearrangement is better as:

$$m = 4t$$

So, rather than divide by a fraction, multiply by its reciprocal when dealing with that layer. E.g. Make t the subject of the formula $\frac{1}{2}t - q = h$:

$$\begin{aligned} \tfrac{1}{2}t - q &= h & \textit{Think of a number, multiply by } \tfrac{1}{2}\textit{, subtract } q \\ \tfrac{1}{2}t &= h + q & \textit{Add } q \textit{ to both sides} \\ t &= 2(h+q) & \textit{Divide both sides by } \tfrac{1}{2}\textit{, that is multiply both by } 2 \end{aligned}$$

Some terms look quite complicated. E.g.

$$\begin{aligned} wxyz &= h \\ wxy &= \frac{h}{z} \\ wx &= \frac{\frac{h}{z}}{y} \\ x &= \frac{\frac{\frac{h}{z}}{y}}{w} \end{aligned}$$

This looks terribly messy with all the fractions stacked up. Instead, read the term in a more efficient way to begin with:

$wxyz$	*I think of a number, multiply by w, multiply by y, multiply by z* — **poor!**
$wxyz$	*I think of a number and multiply it by wyz*

Example. Make t the subject of the following:

$stu - k = m$	*I think of a number, multiply it by su and subtract k*
$stu = m + k$	*Add k to both sides*
$t = \dfrac{m+k}{st}$	

3 More difficult rearrangement (year 9)

When rearranging, there are two layers that are difficult to read and difficult to undo: “taking from” and “dividing into”.

3.1 “Taking from”

Imagine reading $t - mx = k$ in order to make x the subject: I think of a number, multiply it by m and take it from t.
Rather than having to deal with a take from, we swap this difficult layer (when we reach it) for something that is easier to deal with. Since we wish to get rid of a subtraction, we replace it with an addition.

$t - mx = k$	*Add the mx term to both sides*
$t = k + mx$	

This is now easy to read (I think of a number, multiply it by m and add k) and can be undone as readily as before.

Example. Rearrange $k - tp = j$ to make p the subject:

$$\begin{aligned} k - tp &= j && \textit{I think of anumber, multiply it by } t \\ & && \textit{and take it from } k \\ k &= j + tp && \textit{We have dealt with the ``take } tp\textit{'' by} \\ & && \textit{adding to both sides} \\ & && \textit{I think of a number, multiply it by } t \\ & && \textit{and add } j \\ k - j &= tp && \textit{Subtract } j \textit{ from both sides} \\ \frac{k-j}{t} &= p && \textit{Divide both sides by } t \end{aligned}$$

Example. Rearrange $g(t - bm) = k$ to make m the subject:

$$\begin{aligned} G(t - bm) &= kI && \textit{Think of a number, multiply it by } b, \\ & && \textit{take from } t \textit{ and times by } g \\ t - bm &= \frac{k}{G} && \textit{Divide both sides by } g \\ t &= \frac{k}{G} + bm && \textit{Add } bm \textit{ to both sides (we only dealt} \\ & && \textit{with the difficulty as we reached it)} \\ t - \frac{k}{G} &= bm && \textit{Subtract } \tfrac{k}{G} \textit{ from both sides} \\ \frac{t - \frac{k}{G}}{b} &= m && \textit{Divide both sides by } b \end{aligned}$$

Note. In this example it may have been better to expand the brackets first to create a more concise answer:

$$\begin{aligned} G(t - bm) &= k \\ Gt - Gbm &= k \\ Gt &= k + Gbm \\ Gt - k &= Gbm \\ \frac{Gt - k}{Gb} &= m \end{aligned}$$

We could show that the two answers are equal by multiplying top and bottom of the first answer by G.

3.2 "Dividing into"

Imagine reading $\frac{m}{x} = p$ in order to make x the subject: I think of a number, *divide it into* m.

Rather than having to deal with a *divide into*, we swap this difficult layer (when we reach it) for something that is easier to deal with. Since we wish to get rid of a division, we replace it with a multiplication:

Example. Rearrange $\frac{p}{x} - t = q$ to make x the subject:

$$\begin{aligned}
\frac{p}{x} - t &= q && \textit{I think of anumber, divide it into } p \textit{ and take } t\\
\frac{p}{x} &= q + t && \textit{Add } t \textit{ to both sides}\\
p &= x(q + t) && \textit{Deal with the “divide into” by multipying both sides by } x\\
\frac{p}{q + t} &= x && \textit{Divide both sides by } q + t
\end{aligned}$$

You may even get a “take from” and a “divide into” in one formula. Imagine we wished x to be the subject in the following formula:

$$\begin{aligned}
\frac{t}{b - jx} &= k && \textit{Think of a number, multiply by } j\textit{, take it from } b \textit{ and divide into } t\\
t &= k(b - jx) && \textit{Deal with the “divide into” by multiplying by } b - jx\\
t &= bk - jkx && \textit{As we saw before, expanding may make things nicer}\\
t + jkx &= bk && \textit{Deal with the “take from” by adding to}\\
jkx &= bk - t && \textit{Subtract } t \textit{ from both sides}\\
x &= \frac{bk - j}{jk}
\end{aligned}$$

3.3 How are your skills?

Try and follow the steps in this final example, making x the subject:

$$\begin{aligned}
t - \tfrac{1}{3}mx^2 &= k\\
t &= k + \tfrac{1}{3}mx^2\\
t - k &= \tfrac{1}{3}mx^2\\
3(t - k) &= mx^2\\
\frac{3(t - k)}{m} &= x^2\\
x &= \pm\sqrt{\frac{3(t - k)}{m}}
\end{aligned}$$

Final Note. the $\pm$ at the beginning of the above answer is to denote that there are two square roots to any positive value e.g. $\sqrt{25}$ is 5 or -5 since $5 \times 5 = 25$ and $(-5) \times (-5) = 25$. We write ± 5. Try and remember the two answers when using square roots.