Graphs (7–9)

Contents

1 What is a graph?

A graph is a collection of coordinates that all have something in common.

Example. $(3, 2), (3, 3), (3, 17), (3, 100), \ldots$

- the first number is always 3
- i.e. the x-coordinate is always 3

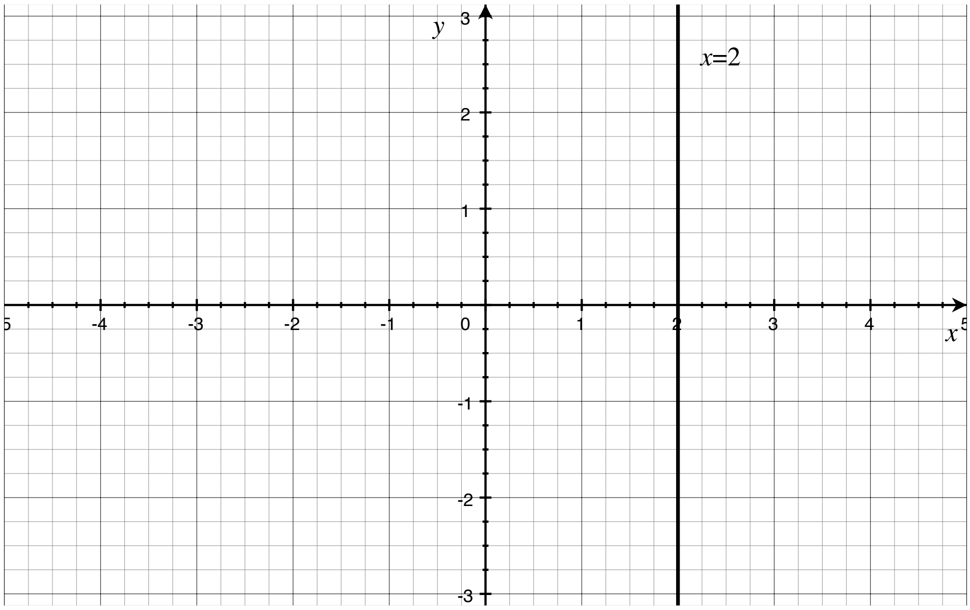
Whatever the coordinates have in common is expressed as an equation. For the example above, the equation is $x = 3$.
What have these coordinates got in common? What is the equation of the graph that they lie on?

$(0, 9), (1, 9), (5, 9), (10, 9), \ldots$	the second number is 9 so $y = 9$
$(1, 1), (2, 2), (3, 3), (4, 4), \ldots$	the numbers are equal so $x = y$
$(1, 4), (2, 3), (3, 2), (4, 1), \ldots$	the pairs add to 5 so $x + y = 5$

2 Graph shapes and plotting graphs

2.1 Horizontal and vertical graphs

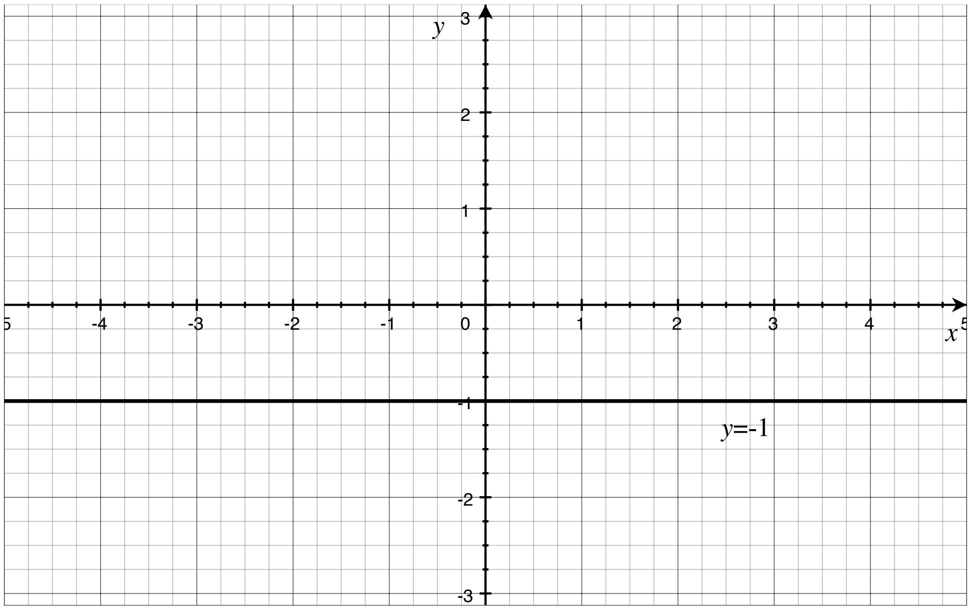
Imagine plotting the coordinates $(2, 0), (2, 1), (2, 2), (2, 3), \ldots$ We see that the first number is always 2, so the equation is $x = 2$.



We see that plotting and joining these points gives a vertical line.

Equations of the form $x =$ *a number* are vertical lines.

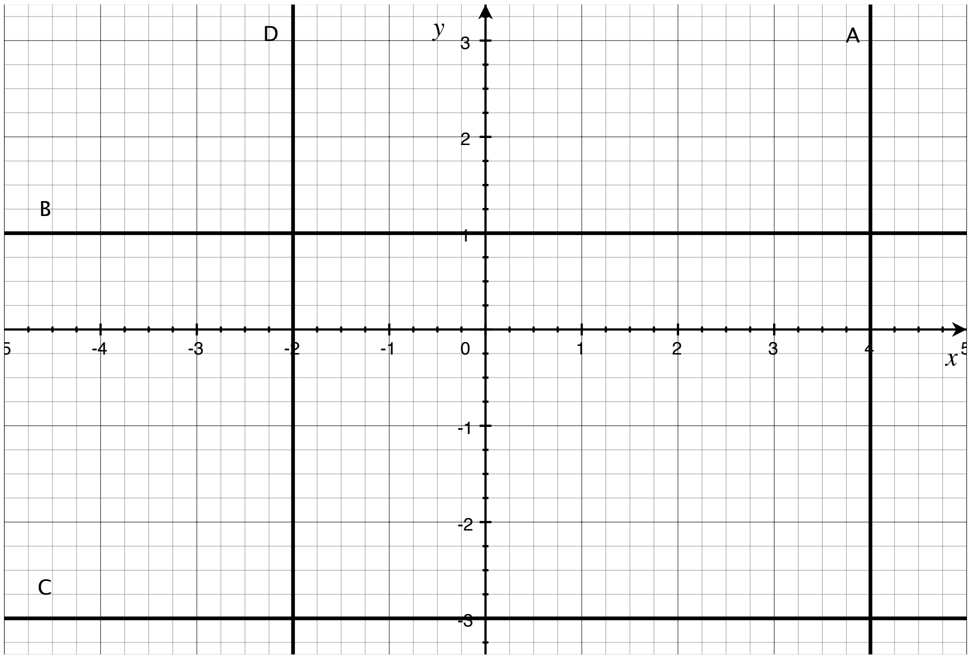
Similarly, if we plot $(1, -1), (2, -1), (3, -1), \ldots$ (that is, the coordinates lying on the line $y = -1$) we get:



We see that plotting and joining these points gives a horizontal line.

Equations of the form $y =$ *a number* are horizontal lines.

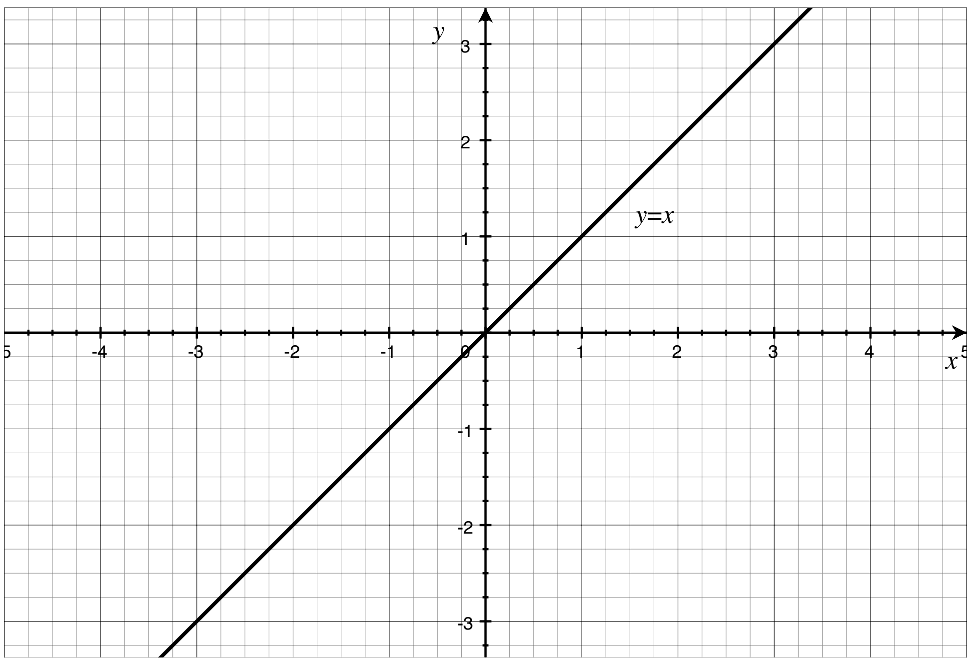
Example. What is the equation of each of the following lines?



- Line A is vertical and passes through 4 on the x-axis, so $x = 4$.
- Line B is vertical and passes through 1 on the y-axis, so $y = 1$.
- Line C is vertical and passes through -3 on the y-axis, so $y = -3$.
- Line D is vertical and passes through -2 on the x-axis, so $x = -2$.

2.2 Diagonal lines

Diagonal lines are formed when an equation has both a y and an x in it. For example $(1, 1)$, $(2, 2)$, $(3, 3)$, $(4, 4), \ldots$ lie on the line $y = x$. You should learn what this line looks like:

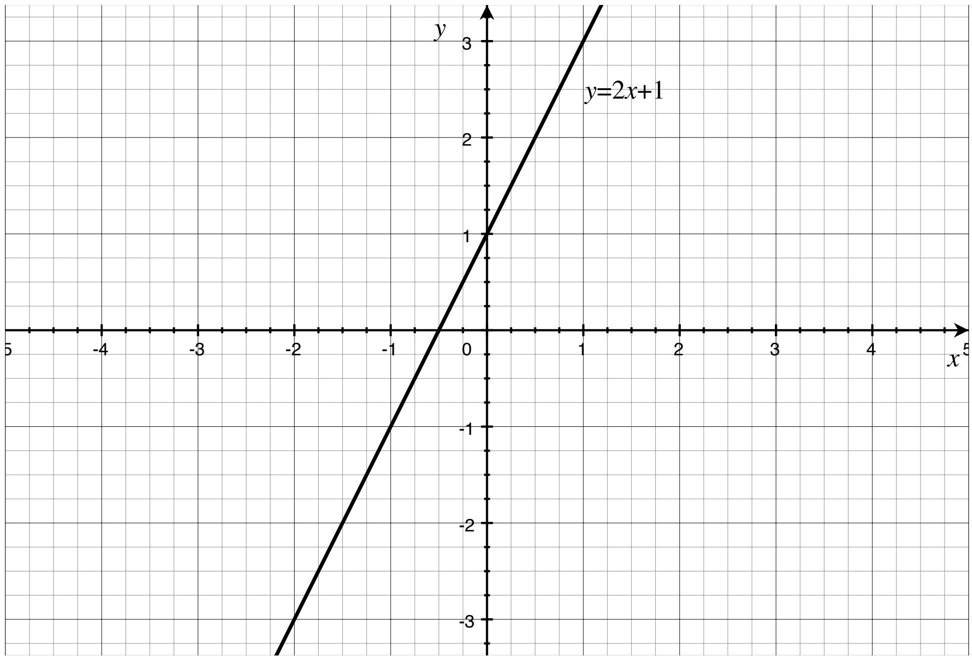


In order to plot a diagonal line, we need to come up with three points that lie on that line (we could get away with two, but the third point is the check) and then join these to create a diagonal line. It is best to come up with these three points in a little x-y table, choosing easy values to substitute such as $x = 1$, $x = 2$ and $x = 3$.

Example. Plot the line $y = 2x + 1$.
When $x = 1$, $y = 2 \times 1 + 1$, so $y = 3$.

x	1	2	3
y	3	5	7

Plot $(1, 3)$, $(2, 5)$, $(3, 7)$ and then join and extend to get $y = 2x + 1$:



Example. Does the point $(10, 12)$ lie on the graph $y = 2x - 5$?
If it does, when we substitute 10 for x we should get 12 for y:

$$\begin{aligned} y &= 2 \times 10 - 5 \\ y &= 20 - 5 \\ y &= 15 \end{aligned}$$

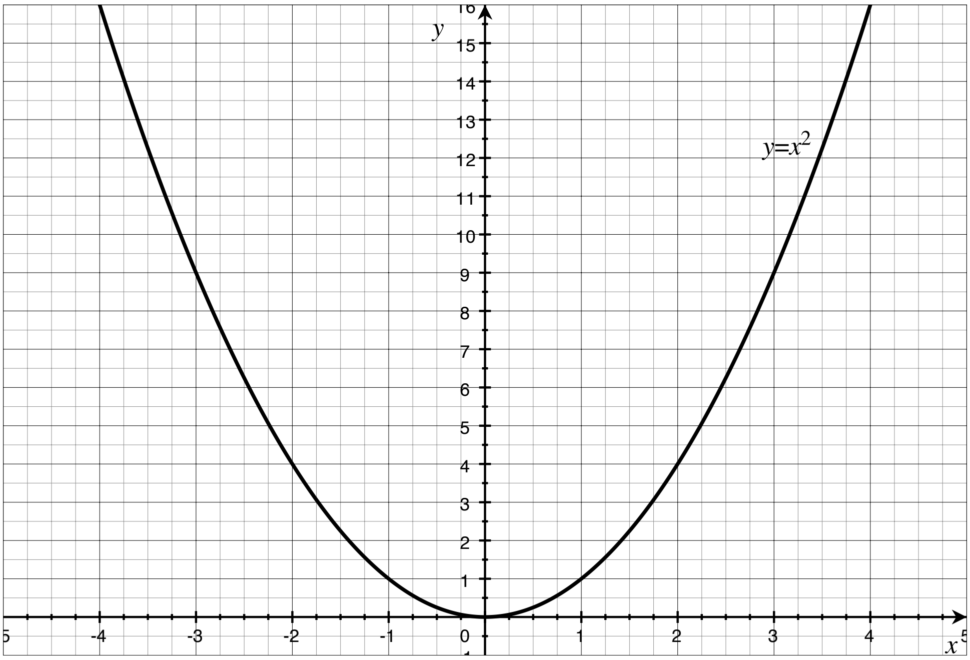
So $(10, 12)$ is not on the line but $(10, 15)$ is.

2.3 Curved graphs

To get a curved graph we need to introduce a “power” into the equation. Consider $y = x^2$ and this table of values that would lie on this graph:

x	-4	-3	-2	-1	0	1	2	3	4
y	16	9	4	1	-0	1	4	9	16

(NB. Squaring a negative gives a positive e.g. $(-3) \times (-3) = 9$)
If we plot these points we get:

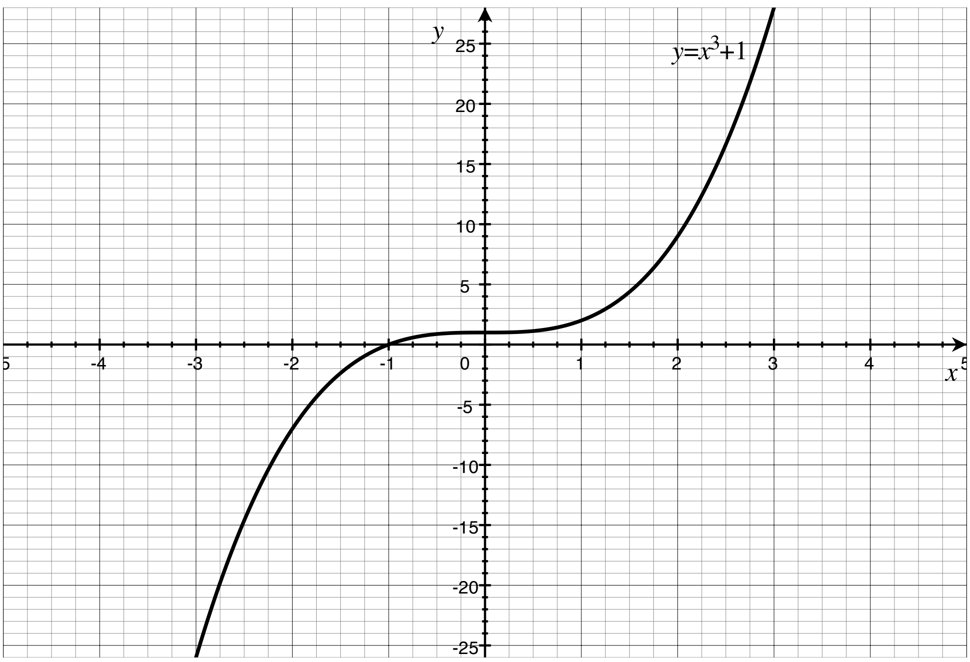


This smooth "U" shape is called a **parabola** and any graph with an x^2 in will be of this form.
To plot a curve, we notice that we need more than 3 points – we need some positive and some negative values to get the full shape of this curve.

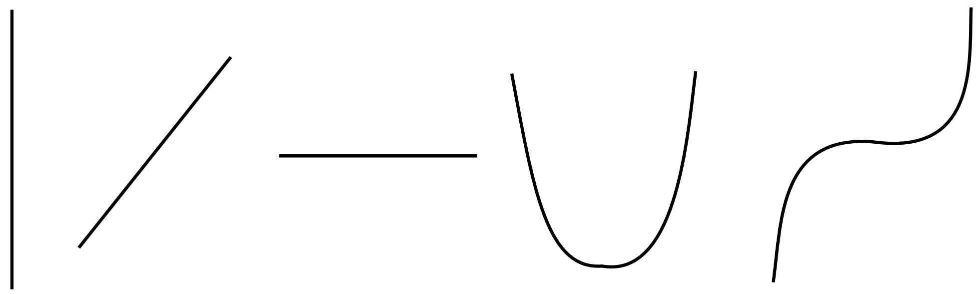
Example. Plot $y = x^3 + 1$.

x	-3	-2	-1	0	1	2	3
y	-26	-7	0	1	2	9	28

(NB. Cubing a negative gives a negative e.g. $(-3)^3 = (-3) \times (-3) \times (-3) = -27$)
The graph below shows the classic "S" shape associated with cubic graphs.



Example. Match each graph shape below with an equation.



$$A : y = x^2 + 1 \quad B : y = 4 \quad C : y = 3x - 1 \quad D : y = 2x^3 \quad E : x = -2$$

- The first is E since it is vertical.
- The second is C since it is diagonal (there is an x and a y in the equation).
- The third is B since it is horizontal.
- The fourth is A since it is a parabola so the equation contains x^2.
- The last is D since it is S-shaped so the equation contains x^3.

3 A deeper look at straight line graphs (year 8 & 9)

We have already seen that diagonal lines have an equation containing an x and a y such as $y = 2x + 1$, $y = 3x - 2$, $y = 5x + 4$ etc. The general equation of a line can be thought of as:

$$y = mx + c,$$

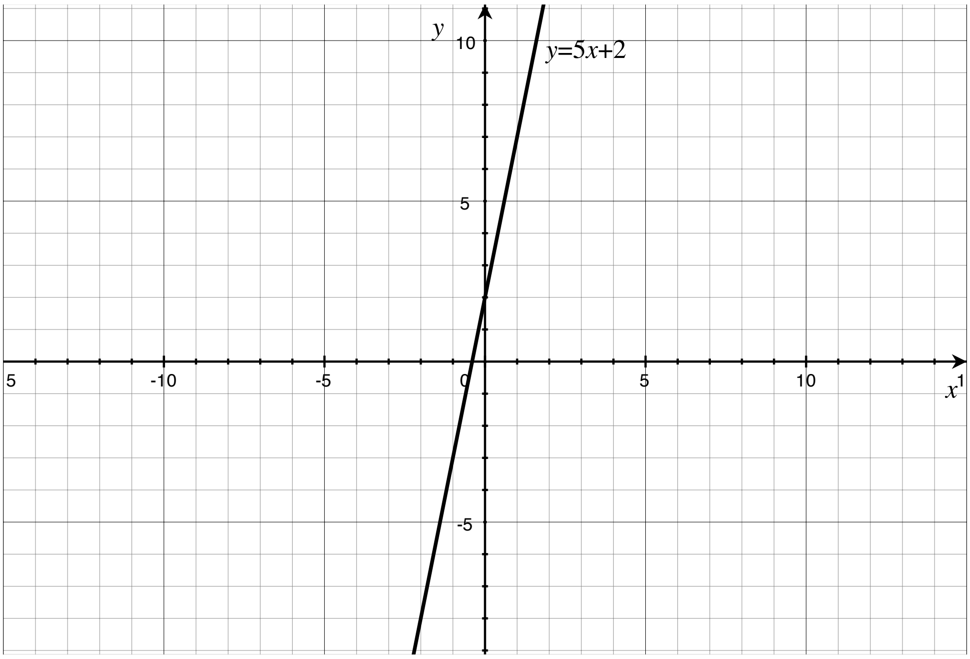
where:

- m **is the gradient:** the higher the value of m, the steeper the line is. If m is negative, the line slopes from top left to bottom right.
- c **is the y-axis intercept:** the value of c tells us where the graph will cross the y-axis.

Let us take the graph $y = 5x + 2$. If we compare it to the form $y = mx + c$, we notice that $m = 5$ and $c = 2$:

- $m = 5$ tells us that the line is quite steep and positive direction;
- $c = 2$ tells us that the line will cut the y-axis at the point $(0, 2)$.

Let us check by plotting the line:



Example. Complete the following table:

Equation	Direction	Gradient	y-axis intercept
$y = 3x + 2$	Positive	3	$(0, 2)$
$y = 4x - 3$	positive	4	$(0, -3)$
$y = -2x - 7$	negative	-2	$(0, -7)$
$y = 5 - 3x$ (think of $y = -3x + 5$)	negative	-3	$(0, 5)$

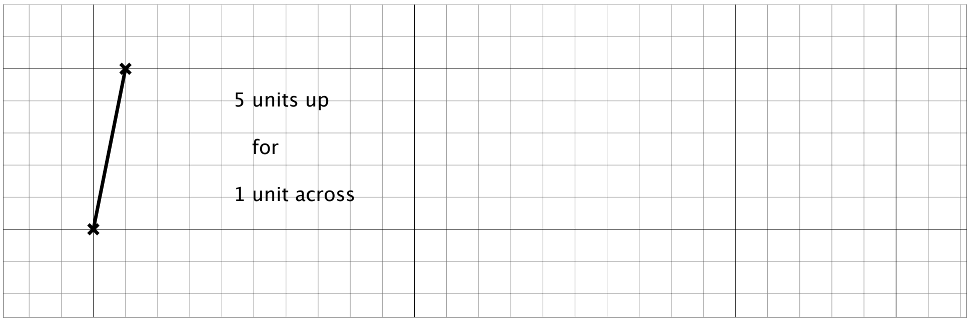
Example. Write down the gradient and y-axis intercept of the diagonal line $2y = 4x - 7$.
The equation must read $y = \dots$ before we can continue. So, we must divide every term by 2 first:

$$\begin{aligned} 2y &= 4x - 7 \\ y &= 2x - 3.5 \end{aligned}$$

Hence, the gradient is 2 and the y-axis intercept is -3.5.

4 A closer look at gradient (year 9)

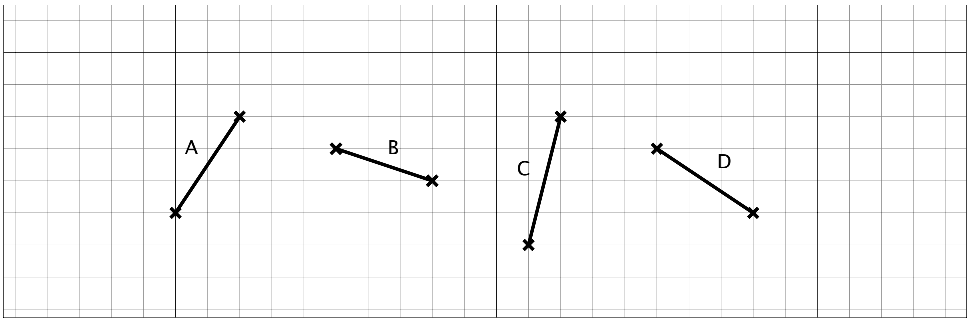
Look at the example drawn above. That is, $y = 5x + 2$. What does a gradient of 5 mean? We notice that this line travels 5 squares up every time it travels 1 square across:



The best way to think about gradient is as a fraction. We know that 5 is really the same as $\frac{5}{1}$ where the top number tells us how many squares to travel up and the bottom number tells us how many squares to travel across. We can remember this as **TUBA**:

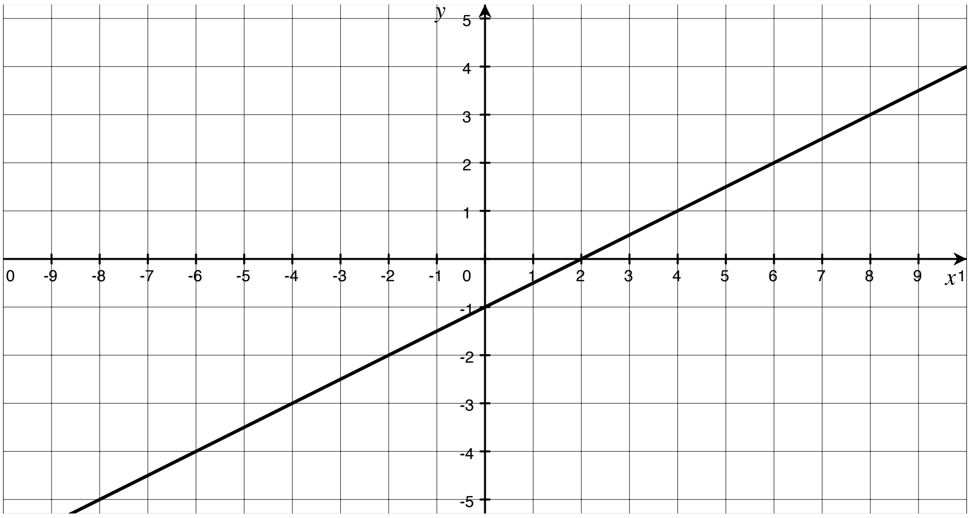
T	top
U	up
B	bottom
A	across

Example. What is the gradient of each of the following line segements?



- line A: this goes 2 across and 3 up so the gradient is $\frac{3}{2}$ or 1.5.
- line B: this goes 3 across and 1 up but in a negative direction so the gradient is $-\frac{1}{3}$.
- line C: this goes 1 across and 4 up so the gradient is $\frac{4}{1}$ or 4.
- line D: this goes 3 across and 2 up but in a negative direction so the gradient is $-\frac{2}{3}$.

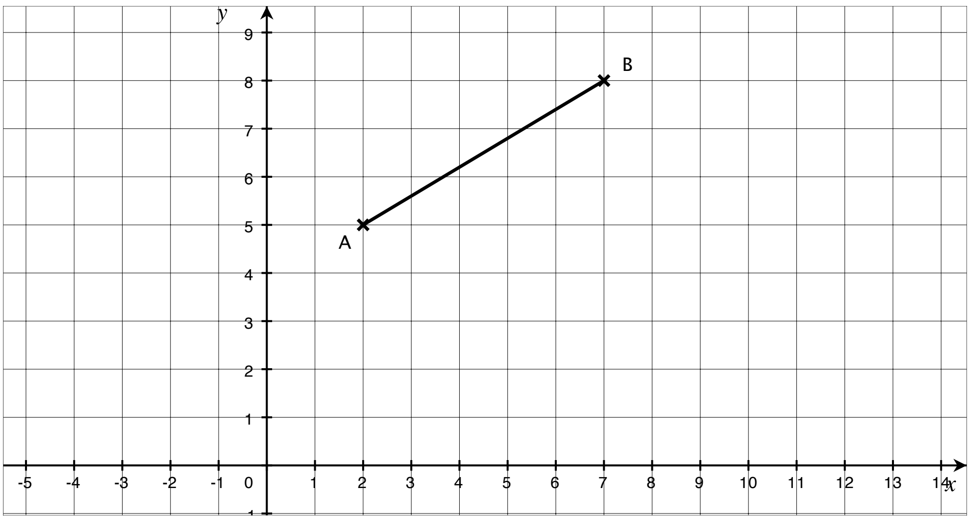
Example. What is the equation of this line?



The graph goes 2 across and 1 up so the gradient is $\frac{1}{2}$. It intersects the y-axis at $(0, -1)$ so the equation is $y = \frac{1}{2}x - 1$.

4.1 What if we don't have squares to count for gradient?

It is possible to work out gradient using a formula. Consider the points with coordinates $(2, 5)$ and $(7, 8)$:



We can see that we need to go 5 across and 3 up to find the gradient of $\frac{3}{5}$. However, we did not need squares to find this. Notice that:

- 5 across is the difference between the x-coordinates, $8 - 2$.
- 3 up is the difference between the y-coordinates, $8 - 5$.

If we consider two points A and B with coordinates called (x_1, y_1) and (x_2, y_2) then:

$$\text{Gradient of } AB = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example. Find the gradient of the line joining the points with coordinates $(9, 7)$ and $(13, -2)$:

$$\begin{aligned} \text{Gradient} = m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - 7}{13 - 9} \\ &= -\frac{9}{4} \text{ or } -2\tfrac{1}{4} \end{aligned}$$

Example. Find the equation of the line joining $(0, 8)$ to $(10, 13)$:

$$\begin{aligned} \text{Gradient} &= \frac{13 - 8}{10 - 0} \\ &= \frac{5}{10} \text{ or } \tfrac{1}{2} \end{aligned}$$

Since the line passes through $(0, 8)$ the y-axis intercept is 8.
Hence,

$$y = \frac{1}{2}x + 8.$$

Example. Find the equation of the line passing through $(4, 9)$ and $(9, 19)$:

$$\begin{aligned} \text{Gradient} &= \frac{19 - 9}{9 - 4} \\ &= \tfrac{10}{5} \text{ or } 2 \end{aligned}$$

As we dont know the y-intercept c, substitute the coordinates of either point either coordinate into the formula $y = mx + c$:

$$\begin{aligned} y &= mx + c \\ y &= 2x + c && \textit{Substitute } m = 2 \\ 9 &= 2 \times 4 + c && \textit{At } (4, 9), x = 4 \textit{ and } y = 9 \\ 9 &= 8 + c \\ 1 &= c \end{aligned}$$

Therefore the line has equation $y = 2x + 1$.