Rules of Indices (8 & 9)

Contents

Introduction

$$\text{base} \rightarrow x^{5} \leftarrow \text{index}$$

There are 3 rules of indices that can be used on expressions that have the same base.

1 Multiplying Indices

Notice that

$$\begin{aligned} p^3 \times p^7 &= (p \times p \times p) \times (p \times p \times p \times p \times p \times p \times p) \\ &= p^{10} \end{aligned}$$

Rather than write this out in long hand each time, we notice that we can simply add the indices. In this case, $3 + 7 = 10$.

Example.

$$\begin{aligned} w^8 \times w^7 &= w^{15} && \textit{Add } 8 \textit{ and } 7. \\ k^{-2} \times k^4 &= k^2 && \textit{Take care adding negatives.} \end{aligned}$$

Notice that $y^6 \times z^3$ is not yz^9. The bases are not equal so we could only simplify this as y^6z^3.

True or false? $3^8 \times 3^4 = 9^{12}$? This is a very common error. The reason why this is actually incorrect is that when using this "rule", the bases remain unchanged. Hence, $3^8 \times 3^4 = 3^{12}$.

2 Dividing Indices

Notice that

$$\begin{aligned} w^9 \div w^4 &= \frac{w \times w \times w \times w \times w \times w \times w \times w \times w}{w \times w \times w \times w} \\ &= w \times w \times w \times w \times w \\ &= w^5 \end{aligned}$$

Rather than write this out in long hand each time, we notice that we can simply subtract the indices. In this case, $9 - 4 = 5$.

Example.

$$\begin{aligned} y^{13} \div y^9 &= y^4 && \textit{Subtract } 9 \textit{ from } 13. \\ h^{-6} \div h^{-3} &= h^{-3} && \textit{As } (-6) - (-3) = -6 + 3 = -3. \end{aligned}$$

3 Indices with brackets

Notice that:

$$\begin{aligned} \left(k^3\right)^2 &= (k \times k \times k) \times (k \times k \times k) \\ &= k^6 \end{aligned}$$

Rather than write this out in long hand each time, we notice that we can simply multiply the indices. In this case, $3 \times 2 = 6$.

Example.

$$\begin{aligned} \left(g^4\right)^7 &= g^{28} \\ \left(m^{-5}\right)^6 &= m^{-30} \end{aligned}$$

4 Summary

When working with expressions with equal bases, we can simplify each of the following by:

multiplying adding the powers/indices

dividing subtracting the indices

brackets multiplying the indices

NB. There is no rule for adding or subtracting since these do not "go" with the multiplication involved in any index. E.g.

$$3^2 + 3^4 = (3 \times 3) + (3 \times 3 \times 3 \times 3) \qquad \textit{A mixture of } + \textit{ and } \times.$$

We can extend these rules to expressions involving coefficients i.e. numbers in front of each term. E.g. In 3×2, 3 is the coefficient of x^2.

$$5p^2 \times 10p^3 = 50p^5$$ *Do* $5 \times 10 = 50$ *and add the indices (Common error* $15p^5$ *— only apply the rule to the indices, not the coefficients)*

$$20b^7 \div 4b^{-2} = 5b^9$$ *Do* $20 \div 4 = 5$ *and subtract the indices.*

$$(5p^3)^2 = 25p^6$$ *Do* $5^2 = 25$ *and multiply the indices.*