# Inequalities (Year 9)

## Contents



## 1 What is an inequality?

An inequality is a mathematical statement containing one of the following signs.

| | |
|---|---|
| $<$ | Less than |
| $\leq$ | Less than or equal to |
| $>$ | Greater than |
| $\geq$ | Greater than or equal to |

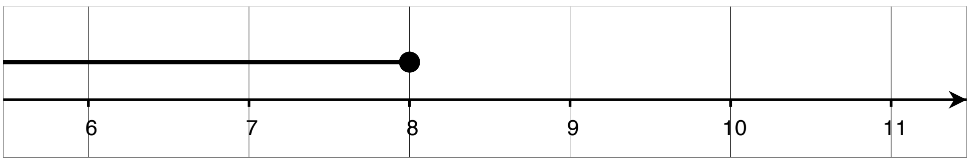
For example, $5 > 3$ is a true inequality. Inequalities occur frequently in real life:

| | |
|---|---|
| You have to be over 18 to buy alcohol | $A > 18$ |
| The lift can only hold 12 passengers | $P \leq 12$ |
| Pop Idol takes contestants between 18 & 25 years inclusive | $18 \leq A \leq 25$ |

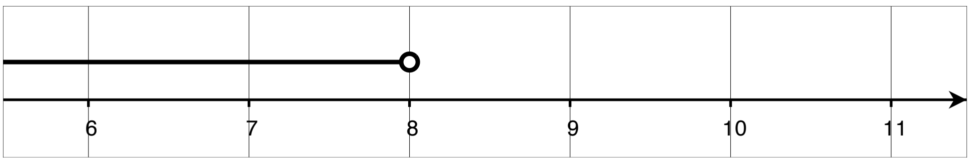
## 2 Displaying inequalities on a number line

If we take an inequality such as $x > 8$, there are many "solutions" to this inequality. For example, $9, 10, 11, 12, 13, 14, 15.2, 16\frac{1}{4}, 100\frac{3}{4}$ etc. For this reason, we show the solution to an inequality on a number line to show this infinite range of answers, rather than writing them all out:

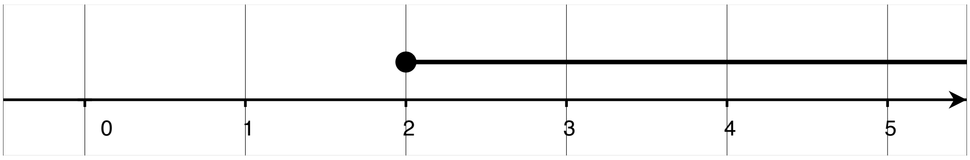
$x \leq 8$



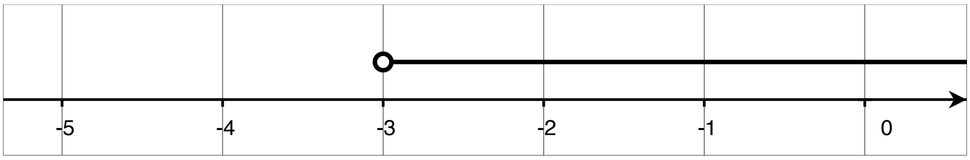

$x < 8$ (this doesnt include 8, it means 7.9999... and below)




$p \geq 2$




$q > -3$




So, we use a solid circle (•) if the number we start from is included in the inequality and a hollow one (○) if it isnt.

# 3 Solving inequalities

It would be easy if we could solve inequalities using the same methods as we do for equations. Let us try performing a mathematical operation to each side of an inequality and see if it remains true:
Consider $4 < 8$ in each case:

| **Operation** | **New Inequality** | **Comment** |
|---|---|---|
| Add $2$ to both sides | $6 < 10$ | True |
| Add $-2$ to both sides | $2 < 6$ | True |
| Subtract $2$ from both sides | $2 < 6$ | True |
| Subtract $-2$ from both sides | $6 < 10$ | True |
| Multiply each side by $2$ | $8 < 16$ | True |
| Multiply each side by $-2$ | $-8 < -16$ | FALSE... only true if we reverse the sign |
| Divide each side by $2$ | $2 < 4$ | True |
| Divide each side by $-2$ | $-2 < -4$ | FALSE ... only true if we reverse the sign |

As the table demonstrates, we can carry out every mathematical operation with positive or negative numbers, except multiplying by a negative and dividing by a negative: in these two cases, we have to swap the inequality sign if we perform these operations.

**Example.** Solve $3x - 9 > 9x + 11$.

$$3x + 9 > 9x - 18$$
$$9 > 9x + 18$$
$$-9 > 9x$$
$$-1 > x$$
$$x > -1 \qquad \text{(we could show this on a number line)}$$

**Example.** Solve $-3y > 12$.

$$\begin{array}{rcl} -3y & > & 12 \\ y & > & -4 \end{array}$$

*We have to divide by* $-3$
*We had to swap the sign*

**Example.** Solve $3x + 9 < 15 < 2x - 1$. In a double inequality, solve each "half" and then combine the answers:

$$\begin{aligned} 3x + 9 &< 15 \\ 3x &< 6 \\ x &< 2 \end{aligned} \qquad \begin{aligned} 15 &< 2x - 1 \\ 16 &< 2x \\ 8 &< x \end{aligned}$$

So, our answer is any number less than 2 and any number over 8.

**Example.** Solve $2x - 10 < 9 < 5x + 14$.

$$\begin{aligned} 2x - 10 &< 9 \\ 2x &< 19 \\ x &< 9.5 \end{aligned} \qquad \begin{aligned} 9 &< 5x + 14 \\ -5 &< 5x \\ -1 &< x \end{aligned}$$

So, our answer is any number greater than -1 and below 9.5. Since this is a continuous range of numbers (and not two separate sections like the previous example), we can write:

$$-1 < x < 9.5$$

## GCSE style question

List the integer values that satisfy $10 < 2x \leq 21$:

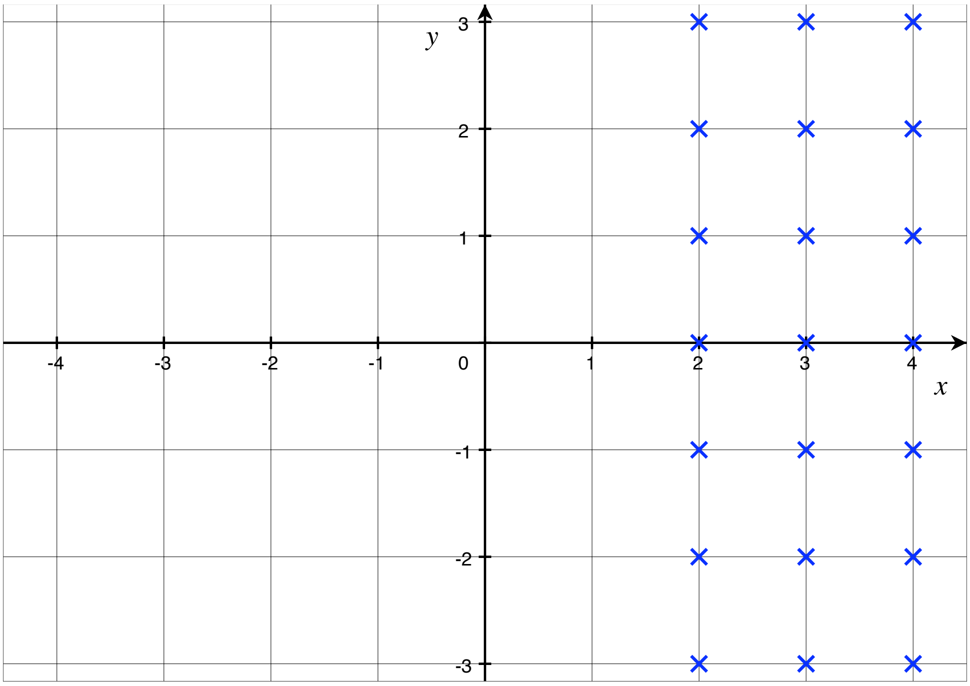
$$\begin{aligned} 10 &< 2x \\ 5 &< x \end{aligned} \qquad \begin{aligned} 2x &\leq 21 \\ x &\leq 11.5 \end{aligned}$$

We want any whole number over $5$ and below or equal to $11.5$. The integers that satisfy this are:

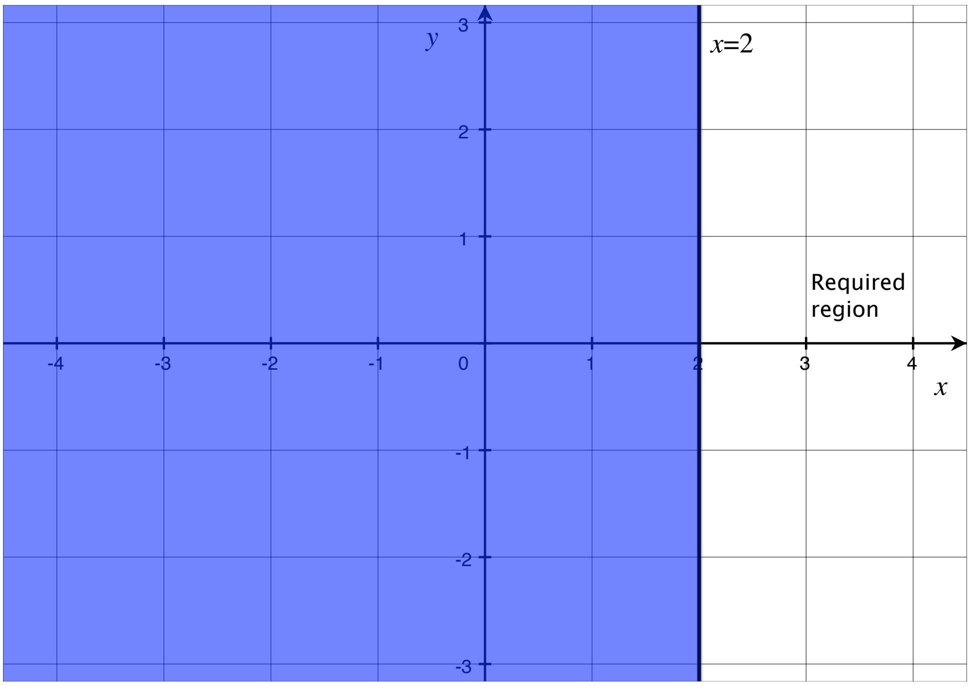
$$6, 7, 8, 9, 10, 11$$

# 4 Showing Inequalities on a graph

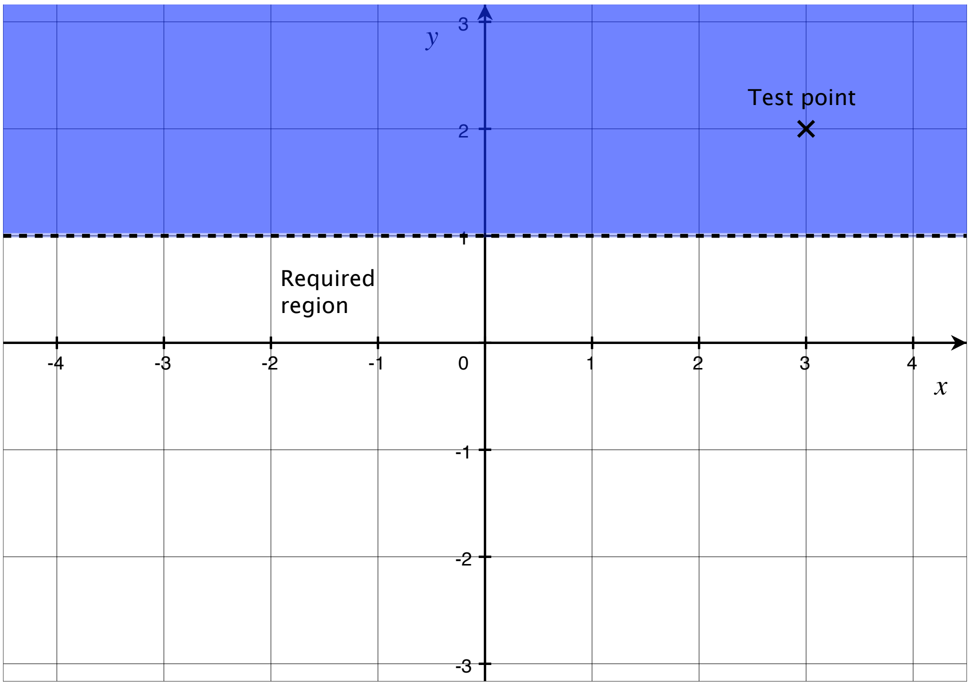
We can represent inequalities in two dimensions using a set of axes. Consider the inequality $x \geq 2$. What we really want here are all the coordinates that we can think of whose first number (the $x$ coordinate) is two or more i.e. $(2, 7), (3, 9), (5, -1) \ldots$ The graphs shows all such coordinates that are integers (i.e. zero or negative/positive whole numbers):




We can see that the coordinates we require create a "blanket" or region. This region starts with the vertical line $x = 2$. In general, we cross out the points we dont want and leave clear what we do:




So, if we wanted to show the inequality $y < 1$, we would: Start by plotting the line $y = 1$ (horizontal, see graph lesson), Shade all the points that we do not want ... if in doubt, check with a "points test" e.g. we know that we do not want $(3, 2)$ since the $y$-coordinate is $2$ and so not below $1$. Since $(3, 2)$ is above our line, we shade out all points above and our required region is all points below:

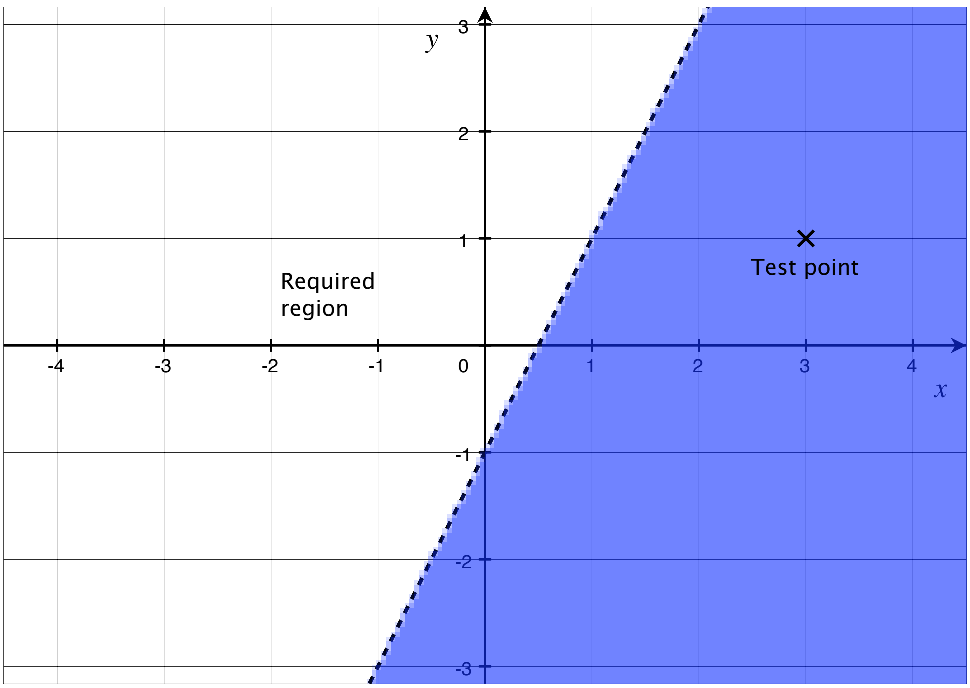



**N.B.** Notice in the example $x \geq 2$ how our graph is a solid line but in the above example, $y < 1$, it is a dotted line. In the second example, we do not want to include "1", just points immediately below it.

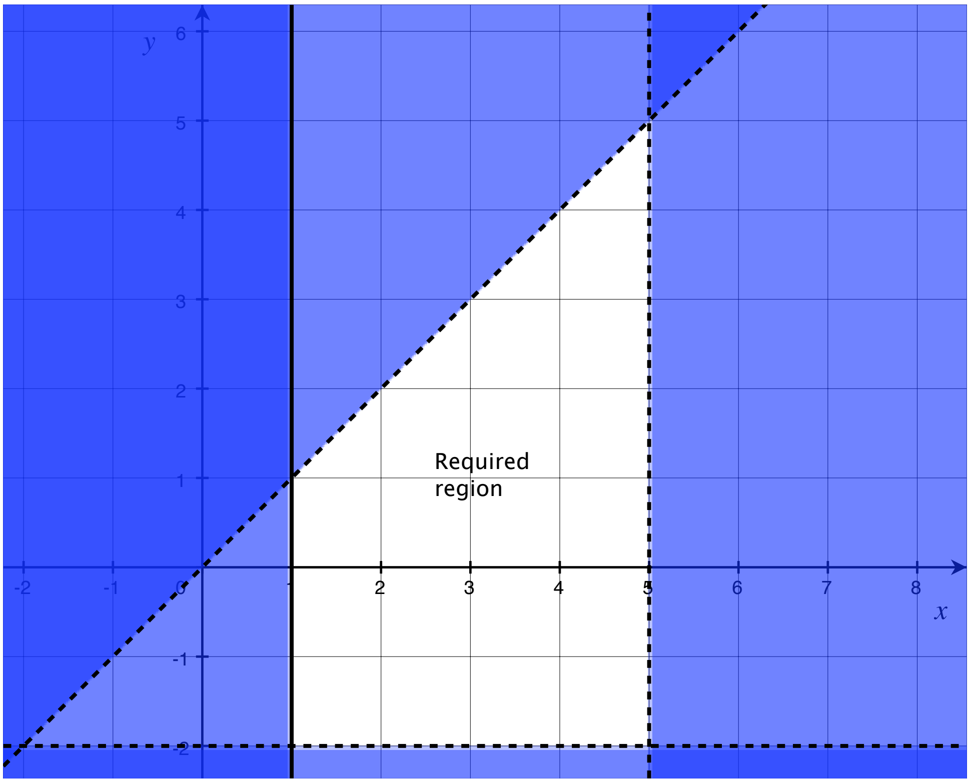
| | |
|---|---|
| $<$ or $>$ | Dotted graph |
| $\leq$ or $\geq$ | Solid graph |

**Example.** Show the region $y > 2x - 1$:

- Draw the graph $y = 2x - 1$ (diagonal line so plot 3 points).
- Points test using $(3, 1)$: is $1 > 3 \times 11$? No, so don't include $(3, 1)$ which is below the line, meaning we want points above the line:



**Example.** What inequalities are shown by the following unshaded region?




The two vertical lines are $x = 1$ (solid) and $x = 5$ (dotted). So they give $x \geq 1$ and $x < 5$, which we can write as $1 \leq x < 5$. The vertical line is $y = -2$ (dotted). This gives $y > -2$. The diagonal line(dotted) passes through $(1, 1), (2, 2), (3, 3) \ldots$ so is $y = x$. It gives us the inequality $y > x$. Hence, the inequalities that give this region are:

$$1 \leq x < 5, \quad y > -2, \quad y > x.$$