Simplifying algebra(7-9)

Contents

Introduction

When expressions are written using the algebra, they may be quite lengthy. We can simplify them in order to make them more concise.

What are terms? Terms are the individual pieces of algebra in any given expression.

$5x + 9$ This has 2 terms: $5x$ and 9.
$4y^3 - 9y + 18z$ This has 3 terms: $4y^3$, $-9y$ and $18z$.

What are like terms? Like terms are terms which contain the same algebra.

$3x$, $9x$, $-10x$ and $17x$ are all like terms.

In the expression $4x + 9y - 2x + 10y$, the terms $4x$ and -$2x$ are like terms as are the terms $9y$ and $10y$.

Some terms look alike but they are not. You may think $3x+9x^2$ contains like terms, but it doesnt. The terms $3x$ and $9x^2$ are similar in appearance but not like terms since the algebra is not identical.

Some terms don't look alike but they are. Consider

$$8ab + 9ba.$$

Since ab means $a \times b$ and ba means $b \times a$, if $a = 3$ and $b = 5$ we know that 3×5 is the same as 5×3. Hence, ab and ba are like terms. Perhaps we should write the letters alphabetically to avoid such confusion.

1 Simplifying when expressions involve addition & subtraction

When expressions involve addition and subtraction, we simplify them by collecting the like terms. It might be good to think of this as counting the like terms. Try and

follow these examples.

$$\begin{aligned} 3x + 9y + 4x + 10y &= 3x + 4x + 9y + 10y \\ &= 7x + 19y \end{aligned}$$

$$\begin{aligned} 10p + 9p^2 + 3p &= 10p + 3p + 9p^2 \\ &= 13p + 9p^2 \end{aligned}$$

$$5x + 9 = 5x + 9 \quad \text{(no like terms to collect)}$$

$$5x + 9x = 14x$$

$$\begin{aligned} 10mn + 7nm &= 10mn + 7mn \\ &= 17mn \end{aligned}$$

If the expression contains negative terms, we must take real care. If you reorder the terms so that like terms are together, make sure you move the negative with the term it is associated with. For example:

$$\begin{aligned} 3x + 10y - 2x - 4y &= 3x - 2x + 10y - 4y \\ &= 1x + 6y \text{ or } x + 6y \end{aligned}$$

$$\begin{aligned} 10m - m^2 + 9m - 4m^2 &= 10m + 9m - m^2 - 4m^2 \\ &= 19m - 5m^2 \end{aligned}$$

Have a look at these classic mistakes that people make when collecting like terms — the mistakes have been corrected for you:

Simplify	**Classic Mistake**	**Correct answer**
$6m + 2$	$8m$	These are not like terms so the expression remains as $6m+2$. Note that $6m + 2m = 8m$
$x^2 + x^2$	x^4	Try to count what you have got. We have an x^2 and then another x^2, so we have $2x^2$
$2a + 3b$	$5ab$	These are not like terms so the expression remains as $2a + 3b$
$2pq + 8qp$	Cannot be simplified	This can be simplified since pq and qp are like terms. The answer is $10pq$
$4m - 9n - 8m + 7n$	$12m + 2n$	Negative signs have not been treated carefully enough: $4m-8m-9n+7n = -4m + 2n$

2 Simplifying when expressions involve multiplication & division

Try really hard not to confuse addition and subtraction with multiplication and division.

With multiplication, we do not need like terms, since the least we can do is to remove the multiplication sign:

$$a \times b = ab$$ *This is simplified even if a & b aren't alike*

When numbers are involved, do these first, then the letters:

$$\begin{aligned} 3m \times 7n &= 21mn \\ 5n \times 2n &= 10n^2 \quad \textit{not } 10n \\ 3p \times 4p^2 &= 3 \times p \times 4 \times p \times p \\ &= 12p^3 \end{aligned}$$

With division, a "divide by" will cancel with a "multiply by":

$$\frac{3m}{3}$$ *I think of a number, multiply it by* 3 *then divide by* 3*. So...*

$$\frac{3m}{3} = m$$

When numbers are involved, do these first, then the letters:

$$\frac{12ab}{3a} = 4a$$ *since* $12 \div 3 = 4$ *and the "*$\times b$*" cancels "*$\div b$*"*

$$\begin{aligned} \frac{10a^2c^3}{2ac} &= \frac{10 \times a \times a \times c \times c \times c}{2 \times a \times c} \\ &= 5ac^2 \end{aligned}$$

If the expression involves addition and division, these do not undo each other:

$\frac{m+3}{3}$ is not m since "$+3$" does not cancel with "$\div 3$"

The only way we can deal with addition/subtraction and division is to read the division line as "all divided by":

$$\frac{6m+9}{3}$$ *i.e.* $6m$ *and* 9 *all have to be divided by* 3*. Hence:*

$$\frac{6m+9}{3} = 2m+3$$