Trial and improvement (8 & 9)

Some equations can be solved by "balancing" whereas others are simply too difficult. We can use *trial and improvement* to take a guess at the solution, see how close we are and then improve on our solution as necessary.

Example. Solve $2x^2 + x = 22.32$

x	x^2	$2x^2$	$2x^2 + x$	Comment
2	4	8	10	Too small
3	9	18	21	Too small
4	16	32	36	Too large
3.5	12.25	24.5	28	Too large
3.3	10.89	21.78	25.08	Too large
3.1	9.61	19.22	22.32	Correct

Notice that:

- $2x^2$ means we need to square first then times by 2 — it is not $(2x)^2$.
- We worked with whole numbers (integers) first of all, sandwiching the answer between 3 and 4 before moving on to decimals.

Not all equations have an exact solution.

Example. Take the equation $x^2 - 3x + 1 = 0$. One of its answers is $2.618033989\ldots$ You would never be asked to find the full answer in this case. Imagine you were asked to solve this equation to 1 decimal place. That is, we need $2.6|18033989\ldots$ rounded to 2.6. Notice how the "1" was the important digit in this case to help us round to 1d.p. That is, if you need an answer to 1d.p., you must work to 2d.p. to know whether to round up or down.
If we were searching for this solution, a good search would look like this:

x	x^2	$3x$	$x^2 - 3x$	$x^2 - 3x + 1$	Comment
1	1	3	-2	-1	Too small
2	4	6	-2	-1	Too small
3	9	9	0	1	Too big
2.5	6.25	7.5	-1.25	-0.25	Too small
2.6	6.76	7.8	-1.04	-0.04	Too small
2.7	7.29	8.1	-0.81	0.19	Too big
2.65	7.0225	7.95	-0.9275	0.0725	Too big

Since 2.65 is too big, we know to *round down* so our answer is 2.6 (1 dp).
Notice:

- We didnt take the answer that was closer out of 2.6 and 2.7 since these are not correct. We dont want an answer that happens to have 1dp, we want the full answer rounded to 1dp for which we will need to know the second decimal place.

- One trial to 2dp is enough in this case.
- **Always sandwich your answers and work to one more decimal place than is needed.**