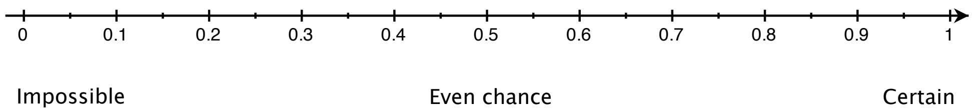
Probability (7–9)

Contents

Introduction

Probability is the chance of an event happening. We would use probability to predict the weather or the horse that is going to win on a race.

Probability can be expressed in words e.g. "likely", but this is very basic. We need to understand the mathematical probability scale. As we see below, this stretches from 0 to 1.



Since a probability must be chosen from a 0 to 1 scale, we use fractions or decimals to represent probability.

1 How to find probabilities

There are 3 key ways to find probabilities, although the last of these is the most common method.

Subjective estimate.

This is really a *guess* based on your best judgement. For example, what is the probability you will watch neighbours tonight?. This will differ for each person, but let's say you generally watch it and only really miss it when you have tennis practice on a Wednesday. So:

$$P(\text{I will watch Neighbours}) = 0.8$$

Relative frequency

This is when the results of an experiment are used to find the chance of an event happening. For instance, what is the probability that the toast and butter you are carrying will land butter side down when dropped? It wouldn't be reliable to only do the experiment, say, two times. The more you can do the experiment, the more reliable your results will be. Say we do the experiment 100 times and the results are as follows

Bread	Frequency
Butter up	34
Butter down	66

So, using this experiment:

$$P(\text{Butter side down}) = \frac{66}{100} \text{ or } 0.66$$

Equally likely events.

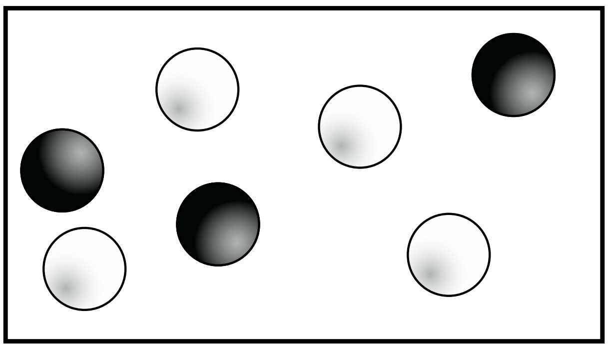
If events are considered to have the same chance of happening e.g. any number on a dice has the same chance of being rolled, then we use the *formula*:

$$P(\text{Event}) = \frac{\text{Number of times the event occurs}}{\text{Total number of events}}$$

Example. Find the probability of getting a 6 on a dice.
As 6 is *one* out of *six* possible options,

$$P(6) = \frac{1}{6}$$

Example. Find the probability of getting a black bead from this box:



There are 3 black beads in the box out of a total of 7 beads, so:

$$P(\text{Black bead}) = \frac{3}{7}$$

2 Combining events

Often we are interested in more than one event happening e.g. what is the chance that I will roll a double six when rolling two dice? There are three key ways that will can combine events to make outcomes and, hence, consider the chance of such an outcome taking place.

2.1 Lists of outcomes

If the situation is straightforward, it may be quick to make a simple list of all of the possible outcomes (don't use this idea in more complex cases as you may miss out some of the options).

Example. your Mum asks what you want for tea — she has fish fingers or sausages and you can have these with potatoes or chips. What is the chance you have fish fingers and chips?

The possible outcomes are: Fish fingers & Potato
Fish fingers & Chips
Sausage & Potato
Sausage & Chips

As there are four possible outcomes, $P(\text{Fish fingers \& Chips}) = \frac{1}{4}$.

2.2 Sample space diagrams

When there are more events to combine to make outcomes, it is more logical to set out the combinations in a sample space diagram. For instance, what is the possibility that I will roll a double six when rolling two dice?

	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	$\boxed{6, 6}$

We can see there are 36 outcomes and a double 6 (framed above) only occurs once. Therefore

$$P(\text{Double six}) = \frac{1}{36}$$

This isn't very likely so you understand why it is difficult to get started in a board game where a double six is required before you make your first move!

2.3 Tree diagrams (8 & 9 only)

Sample space diagrams are restricted to two events e.g. rolling two dice and also to equally likely events. To overcome these problems, we can use a tree diagram instead.
Before we understand tree diagrams, we need to appreciate two rules of probability.

The OR rule

If you roll a dice, you know that that

- $P(\text{6 on a dice}) = \frac{1}{6}$ (as it can only be 6)
- $P(\text{odd number on a dice}) = \frac{3}{6}$ (as it can be 1, 3 or 6)
- $P(6$ **OR** an odd number on a dice$) = \frac{4}{6}$ (as it can be 1, 3, 5 or 6)

We notice that we could add $\frac{1}{6}$ and $\frac{3}{6}$ in order to get $\frac{4}{6}$. That is,

$$P(A \textbf{ OR } B) = P(A) + P(B)$$

i.e. the word "or" means add in probability.

The AND rule

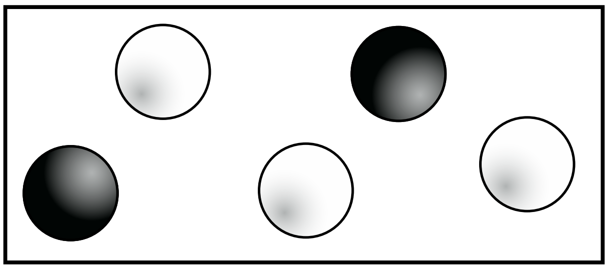
- $P(\text{6 on a dice}) = \frac{1}{6}$
- $P(\text{Double six}) = P(6 \textbf{ AND } 6) = \frac{1}{36}$ (see above)

We notice that we could multiply $\frac{1}{6}$ and $\frac{1}{6}$ in order to get $\frac{1}{36}$. That is,

$$P(A \textbf{ AND } B) = P(A) \times P(B)$$

i.e. the word "and" means multiply in probability.

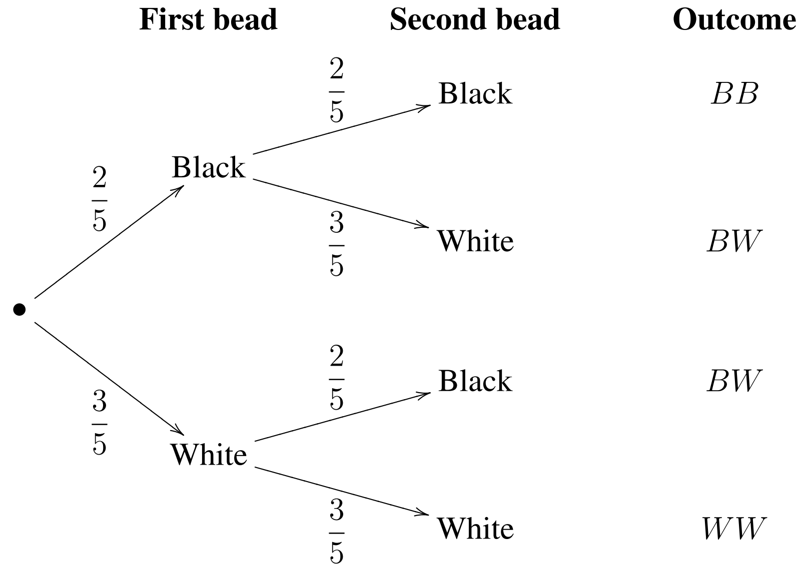
So, let's have a look at a tree diagram. Imagine we have a box containing 3 white and two black counters:



Imagine putting in your hand and picking a counter, noting its colour and then replacing it. This is then repeated. What are the different outcomes and what is the chance of each such outcome?

N.B. You cannot say the outcomes are BB, BW, WB or WW each with a chance of $\frac{1}{4}$ since it is going to much more likely that we get WW, say, than BB since there are more whites. This shows why a list or even a sample space diagram would be no good here.

A tree diagram would look like this:



N.B. Notice how BW is different to WB.
Now we can find probabilities:

- The probability that I pick two black beads:

$$\begin{aligned} P(\text{Two blacks}) &= P(BB) \\ &= \frac{2}{5} \times \frac{2}{5} \\ &= \frac{4}{25} \end{aligned}$$

- The probability that I pick one bead of each colour:

$$\begin{aligned} P(\text{One of each colour}) &= P(BW \textbf{ OR } WB) \\ &= P(BW) + P(WB) \\ &= \frac{2}{5} \times \frac{3}{5} + \frac{3}{5} \times \frac{2}{5} \\ &= \frac{6}{25} + \frac{6}{25} \\ &= \frac{12}{25} \end{aligned}$$

- The probability that I pick at least one black bead:

$$\begin{aligned} P(\text{At least one black}) &= P(BW \textbf{ OR } WB \textbf{ OR } BB) \\ &= \frac{2}{5} \times \frac{3}{5} + \frac{3}{5} \times \frac{2}{5} + \frac{2}{5} \times \frac{2}{5} \\ &= \frac{16}{25} \end{aligned}$$