Fraction ordering and calculations (7–8)

Contents

1 Vocabulary

$\frac{3}{5}$ This is a **proper fraction** with a **numerator** of 3 and a **denominator** of 5

$\frac{7}{5}$ This is an **improper fraction**. It can be changed to a mixed number by noticing that 5 goes into 7 once, with 2 left over i.e. $1\frac{2}{5}$.

$3\frac{2}{5}$ This is a **mixed number**. It can be changed to an improper fraction by doing $(3 \times 5) + 2$ to get $\frac{17}{5}$.

$\frac{6}{10}$ and $\frac{3}{5}$ These are **equivalent fractions**, but only $\frac{3}{5}$ is in its **simplest terms**. We can convert between fractions by multiplying or dividing the numerator and denominator by the same amount (in this case by two).

2 Ordering fractions

It is easy to see that $\frac{3}{5}$ is smaller than $\frac{4}{5}$ since they are the same type of fraction. However, it is not so easy to see which is largest between $\frac{7}{8}$ and $\frac{6}{7}$. To decide this, we should convert each fraction into the same type i.e. ones with the same denominator:

$$\frac{7}{8} = \frac{7 \times 7}{8 \times 7} = \frac{49}{56} \qquad \frac{6}{7} = \frac{6 \times 8}{7 \times 8} = \frac{48}{56}$$

We can now see that the first fraction is slightly larger (by $\frac{1}{56}$ only!).

3 Fraction calculations

3.1 Fraction of a quantity

If we wanted to find $\frac{3}{4}$ of 20, we would start by finding one quarter (by dividing by 4) then finding three of these:

$$\begin{aligned}\frac{3}{4} \text{ of } 20 &= 3 \times (20 \div 4) \\ &= 3 \times 5 \\ &= 15\end{aligned}$$

This can be extended to more difficult values. For instance,

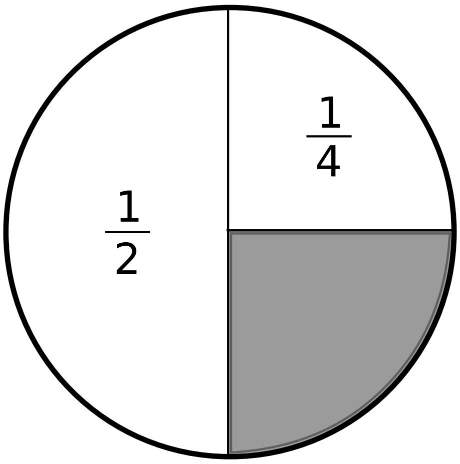
$$\begin{aligned}\frac{4}{17} \text{ of } 85 &= 4 \times (85 \div 17) \\ &= 4 \times 5 \\ &= 20\end{aligned}$$

3.2 Adding and subtracting fractions

If we think about this sum, we can see it is clearly wrong:

$$\frac{1}{2} + \frac{1}{4} = \frac{2}{6}$$

By imagining, say, a cake, we can see that $\frac{1}{2}$ added to $\frac{1}{4}$ gives $\frac{3}{4}$:



That is, we cannot add or subtract fractions by simply adding the tops and adding the bottoms. However, we know this to be true:

$$\frac{1}{4} + \frac{1}{4} = \frac{2}{4}.$$

So, as long as the fractions of from the same family, we simply add the "tops"(numerators). If they aren't from the same family, we have to convert them first:

$$\begin{aligned}\frac{2}{5}+\frac{1}{3} &= \frac{2\times 3}{5\times 3}+\frac{1\times 5}{3\times 5}\\ &= \frac{6}{15}+\frac{5}{15}\\ &= \frac{11}{15}\end{aligned}$$

If the numbers are mixed, we need to convert to top heavy to begin with:

$$\begin{aligned}3\frac{1}{3}-2\frac{2}{7} &= \frac{10}{3}-\frac{16}{7}\\ &= \frac{10\times 7}{3\times 7}-\frac{16\times 3}{7\times 3}\\ &= \frac{70}{21}-\frac{48}{21}\\ &= \frac{22}{21}\\ &= 1\frac{1}{21}\end{aligned}$$

3.3 Multiplying and dividing fractions

If we think about what "half of a half" is, we can see this is a quarter. That is:

$$\frac{1}{2}\text{ of }\frac{1}{2}=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}.$$

Hence, we notice that to multiply fractions we simply multiply the numerators and the denominators ("tops" and "bottoms"). For example,

$$\frac{2}{7}\times\frac{3}{5}=\frac{6}{35}.$$

If we look at the following example, we see that we may need to cancel the answer:

$$\begin{aligned}\frac{2}{7}\times\frac{3}{4} &= \frac{6}{28}\\ &= \frac{3}{14}.\end{aligned}$$

Rather than cancelling at the end, we could try to cancel top to bottom before multiplying:

$$\begin{aligned}\frac{2}{7}\times\frac{3}{4} &= \frac{\not{2}^{1}}{7}\times\frac{3}{\not{4}_{2}}\\ &= \frac{1}{7}\times\frac{3}{2}\\ &= \frac{3}{14}.\end{aligned}$$

Think about the expression $50 \div \frac{1}{2}$. It is very tempting to think the answer is 25, but we see that $\frac{1}{2}$ "fits into" 50 one hundred times exactly. That is:

$$50 \div \frac{1}{2} = 100.$$

We notice, then, that $50 \div \frac{1}{2}$ is equivalent to 50×2.
So, to divide by a fraction, we multiply by the second fraction "flipped over". That is, we multiply by the **reciprocal** of the second fraction. E.g.

$$\begin{aligned} \frac{2}{9} \div \frac{3}{5} &= \frac{2}{9} \times \frac{5}{3} \\ &= \frac{10}{27}. \end{aligned}$$