Zero, negative & fractional indices (years 8 & 9)

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1 Indices

This is the collective name given to any power. For instance, in 3^4, the "4" is the *power* or *index*. To work out any index, we multiply the base number by itself that many times. For instance:

$$34 = 3 \times 3 \times 3 \times 3 = 81$$

On your calculator, use the $\boxed{x^y}$ or $\boxed{\wedge}$ button to work out powers.

2 Zero & Negative Indices

Consider the following number line showing powers of 3:

3^1	3^2	3^3	3^4	3^5
3	9	27	81	243

Notice that as we move right along the number line, the index increases by one and we multiply by 3. So, we could extend this number line to the left by decreasing the index by one and by dividing by 3 to get:

3^{-5}	3^{-4}	3^{-3}	3^{-2}	3^{-1}	3^0	3^1	3^2	3^3	3^4	3^5
$\frac{1}{243}$	$\frac{1}{81}$	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	27	81	243

Notice the following.

- Any number to the power of zero is 1:

$$4^0 = 1 \qquad 15^0 = 1 \qquad (-7)^0 = 1$$

- To work out a negative power, it is the same answer as the positive power but "flipped" over. That is, it is the *reciprocal* of the positive answer:

$$4^{-2} = \frac{1}{16} \qquad 2^{-3} = \frac{1}{8} \qquad \left(\frac{3}{4}\right)^{-2} = \frac{16}{9} \text{ or } 1\frac{7}{9}$$

LEARN THESE TWO POINTS BY HEART!!

Common error It is very tempting to say $3^0 = 0$, but you need to remember that anything to the power of zero is 1.

3 Fractional Indices

If we use the power key on our calculator, we notice the following:

$$25^{\frac{1}{2}} = 5 \qquad 36^{\frac{1}{2}} = 6 \qquad 100^{\frac{1}{2}} = 10$$

It seems that raising to the power of $\frac{1}{2}$ is the same as taking the square root.
What about raising to the power of $\frac{1}{3}$? What could this mean? Lets try some:

$$8^{\frac{1}{3}} = 3 \qquad 64^{\frac{1}{3}} = 4 \qquad 1,000^{\frac{1}{3}} = 10$$

It seems that raising to the power of $\frac{1}{3}$ is the same as taking the cube root.

LEARN | "To the power of $\frac{1}{2}$" is the same as "square root"
"To the power of $\frac{1}{3}$" is the same as "cube root"

Example. Study the following:

$49^{\frac{1}{2}} = 7$ *since this is the square root of 49*

$169^{\frac{1}{2}} = 13$ *since this is the square root of 169*

$125^{\frac{1}{3}} = 5$ *since this is the cube root of 125*

4 Combining fractional and negative indices

Follow these examples:

$25^{-\frac{1}{2}} = \ldots$ *the* $\frac{1}{2}$ *tells us to square root 25*

$= \ldots$ *and the negative power means*

$= \ldots$ *that we have to do the reciprocal of this (flip it over!)*

$= \frac{1}{5}$

$64^{-\frac{2}{3}} = \ldots$ *the* $\frac{1}{3}$ *tells us to cube root 64 (which is 4)*

$= \ldots$ *the 2 tells us to square this (which is 16)*

$= \ldots$ *and the negative power means taking the reciprocal*

$= \frac{1}{16}$

5 Key facts

- Anything to the power of zero is 1.
- A negative power tells you to "flip" (do the reciprocal) of the answer if the power was positive.
- Rasing to the power of $\frac{1}{2}$ means taking the square root and to the power of $\frac{1}{3}$ means taking the cube root.