Negative numbers (7)

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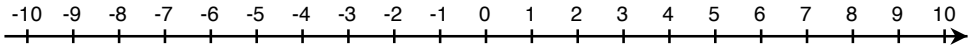
Introduction

Negative numbers are lower than zero on the number line: they are used, for example, to measure really cold temperatures. The lowest temperature ever recorded on earth was $-89°$C in Antarctica!!



The further than negative number is from zero, the smaller it will be. Since -9 is further from zero than -3, then -9 is smaller than -3. We write:

$$-9 < -3$$

1 Adding and subtracting with negative numbers

If we add or subtract a positive number, we should use the number line to work out our answer. Consider $-5+7$.

-5	+	7
Starting position	Move right	This many places

Hence, $-5+7=2$.

Example. Check you understand these examples by referring to the number line:

$$\begin{aligned} -2+8 &= 6 \\ -4+3 &= -1 \\ 57 &= -2 \\ -34 &= -7 \end{aligned}$$

If we add or take a *negative* number, we need to extend the patterns that we know already to see how these will work:

$$\begin{aligned} 3+4&=7 & 3-4&=-1\\ 3+3&=6 & 3-3&=0\\ 3+2&=5 & 3-2&=1\\ 3+1&=4 & 3-1&=2\\ 3+0&=3 & 3-0&=3\\ 3+(-1)&=2 & 3-(-1)&=4\\ 3+(-2)&=1 & 3-(-2)&=5\\ 3+(-3)&=0 & 3-(-3)&=6\\ 3+(-4)&=-1 & 3-(-4)&=7 \end{aligned}$$

- Notice that $3+(-4)=-1$ — this is the same as $3-4$.
- Notice also that $3-(-4)=7$ — this is the same as $3+4$.

The trick is to deal with the two signs that are together in the middle of the sum. If it is a positive and a negative, this can simply be replaced with a negative. If it is two negatives, this can simply be replaced with a positive. We would then carry on using the number line as before. E.g.

$5+(-9)$	the "$+$" and "$-$" are replaced with a "$-$" giving $5-9=-4$
$6-(-2)$	the "$-$" and "$-$" are replaced with a "$+$" giving $6+2=8$
$(-7)-(-3)$	the "$-$" and "$-$" are replaced with a "$+$" so $(-7)+3=-4$

Notice in the last example that the negative at the front is not replaced. We are only looking at the two signs that are together in the middle of the sum.

Memory trick

If you think of a negative as a "bad" and a positive as a "good", then:

$-/+$ replaced by $-$	if something bad happens to somebody good, that makes you feel bad
$+/-$ replaced by $-$	if something good happens to somebody bad, that makes you feel bad
$-/-$ replaced by $+$	if something bad happens to somebody bad, that makes you feel good

2 Multiplying and dividing with negatives

You mustn't confuse adding/subtracting with multiplying/dividing. Try to keep the methods separate in your head. For multiplication and division, the patterns in this

times table grid will help us:

$\times$	-2	-1	0	1	2
-2	4	2	0	-2	-4
-1	2	1	0	-1	-2
0	0	0	0	0	0
1	-2	-1	0	1	2
2	-4	-2	0	2	4

We can see that:

$$\begin{aligned}\text{Positive} \times \text{Positive} &= \text{Positive}\\ \text{Positive} \times \text{Negative} &= \text{Negative}\\ \text{Negative} \times \text{Positive} &= \text{Negative}\\ \text{Negative} \times \text{Negative} &= \text{Positive}\end{aligned}$$

Since multiplication and division are closely linked, the same is true for division. We can use the little memory aid with "good" and "bad" to help us to remember this.

Example.

$$(-3) \times (-7) = 21$$

First do $3 \times 7 = 21$*, then remember that neg* $\times$ *neg* $=$ *pos.*

$$\begin{aligned}(-30) \div 5 &= -6\\ (-4) \times 7 &= -28\\ 36 \div (-6) &= -6\\ (-144) \div (-12) &= 12\end{aligned}$$

This will allow us to square and cube negatives too.

$$\begin{aligned}(-3)^2 &= (-3) \times (-3) && \textit{You always get a positive when you square a negative}\\ &= 9\\ (-4)^3 &= (-4) \times (-4) \times (-4) && \textit{You always get a negative when you cube a negative}\\ &= -64\end{aligned}$$

Challenge. Can you see why $\sqrt{100}$ is either -10 or 10 but $\sqrt{-100}$ cannot be done?