Percentages (years 7–9)

Contents

1 Mental Percentages (all years)

Simple percentages (1%, 20%, 90%, 50%, 35%, ...) can be easily calculated in your head without having to use a calculator. It is always worth starting with key percentages (10% or 50%, say) that are easy to find and using multiples or fractions of these combined to make the percentage that you want.

N.B. To find 10%, simply divide the number by 10. E.g.

$$10\% \text{ of } 360 = 36 \qquad 10\% \text{ of } 7,000 = 700 \qquad 10\% \text{ of } 42 = 4.2$$

Example. Suppose we wanted to find 35% of 900:

$$\begin{aligned} 10\% \text{ of } 900 &= 90 \\ \text{So, } 5\% \text{ of } 900 &= 45 && \textit{(by halving 10\%)} \\ \text{So, } 35\% &= 90 + 90 + 90 + 45 && \textit{(three 10\% + one 5\%)} \\ &= 315 \end{aligned}$$

Always make sure that your answer is reasonable. Since 35% is under half (i.e. 50%), it seems reasonable that 35% of 900 = 315.

Example. Suppose we wanted to find 95% of £420:

$$\begin{aligned} 100\% \text{ of } 420 &= 420 \\ 10\% \text{ of } 420 &= 42 \\ So 5\% \text{ of } 420 &= 21 \\ Now, 95\% &= 100\% - 5\% = 42021 = \pounds 399 \end{aligned}$$

If you dont like this method, you could try the following:

$$\begin{aligned} 35\% \text{ of } 900 &= \frac{35}{100} \times 900 \\ &= 35 \times 9 && \textit{(Cancel 900 with 100)} \\ &= 315 \end{aligned}$$

2 Calculator Percentages

2.1 Year 7 method

More difficult percentages should be found using a calculator: simply convert the sentence in English to one that makes sense in mathematical language. E.g.

From English…	52%	of	£346
…to Maths	$\frac{52}{100}$	$\times$	346

$$\begin{aligned} \text{So, } 52\% \text{ of } \pounds 346 &= \frac{52}{100} \times 346 \\ &= \pounds 179.92 \end{aligned}$$

This can then be extended to calculations involving increase and decrease. For example: A man receives £320 a month. His monthly salary is set to rise by 7%. What will his new pay be?

$$\begin{aligned} 7\% \text{ of } 320 &= \frac{7}{100} \times 320 && \textit{First we have to find 7\% of £320.} \\ &= 22.4 \\ \text{New salary} &= 320 + 22.4 && \textit{Then add this on.} \\ &= \pounds 344.40 \end{aligned}$$

A tv costs £300 but is to be reduced by 12% in a sale. What will its sale price be?

$$\begin{aligned} 12\% \text{ of } 300 &= \frac{12}{100} \times 300 && \textit{First we have to find 12\% of £300.} \\ &= 36 \\ \text{Sale price} &= 300 - 36 && \textit{And then subtract this.} \\ &= \pounds 264 \end{aligned}$$

Compound percentages

Sometimes percentages are added on many times e.g. a bank giving interest year after year. In this case, you must find the percentage of each new amount and keep adding or taking this.

Example. A man puts £400 into a savings account that pays 3% interest each year. How much will he have after 2 years?

Do not think 3% twice is the same as 6% since the savings will have increased after one year so we will be finding 3% of a larger amount.

$$\begin{aligned} 3\% \text{ of } 400 &= \frac{3}{100} \times 400 \\ &= 12 && \textit{First year interest} \\ \text{New savings} &= 400 + 12 \\ &= £412 && \textit{After one year} \\ 3\% \text{ of } 412 &= \frac{3}{100} \times 412 \\ &= 12/36 && \textit{Second year interest} \\ \text{New savings} &= 412 + 12.36 \\ &= £412.36 && \textit{After two years} \end{aligned}$$

2.2 Year 8 & 9 method

Notice how the above example took lots of steps ... we had to find 3%, add it on, find 3% again and add it on again! Wouldnt like be simpler if we could do this in one step? Well, we can with *decimal multipliers*.

You need to know how to use decimal multipliers in **percentage of**, **percentage increase** and **percentage decrease** calculations. See if you can follow this example:

$$\begin{aligned} 40\% \textbf{ of } \ldots &= \frac{40}{100} \times \ldots \\ &= 0.4 \times \ldots && \textit{The decimal multiplier for 40\% of is 0.4} \end{aligned}$$

$$\begin{aligned} 40\% \textbf{ increase } \ldots &= 140\% \textbf{ of } \ldots \\ &= \frac{140}{100} \times \ldots \\ &= 1.4 \times \ldots && \textit{The decimal multiplier for 40\% increase is 1.4} \end{aligned}$$

$$\begin{aligned} 40\% \textbf{ decrease } \ldots &= 60\% \textbf{ of } \ldots \\ &= \frac{60}{100} \times \ldots \\ &= 0.6 \times \ldots && \textit{The decimal multiplier for 40\% decrease is 0.6} \end{aligned}$$

The following table shows further decimal multipliers. Can you understand how they were found?

Percentage	**Of...**	**Increase...**	**Decrease...**
70%	0.7	1.7	0.3
2%	0.02	1.02	0.98
24%	0.24	1.24	0.76

These can then be used to perform calculations in one step:

Example. A man wins £25,400 on the lottery and gives 4% to charity. How much does he give to charity?

$$\begin{aligned}\text{Amount to charity} &= 0.04 \times £25,400 \\ &= £1016\end{aligned}$$

Example. A house costing £112,000 is renovated and its value increases by 60%. How much is it worth now?

$$\begin{aligned}\text{New price} &= 1.6 \times 112,000 \\ &= £179,200\end{aligned}$$

Example. A pair of £40 jeans are reduced by 13% in a sale. What is their sale price?

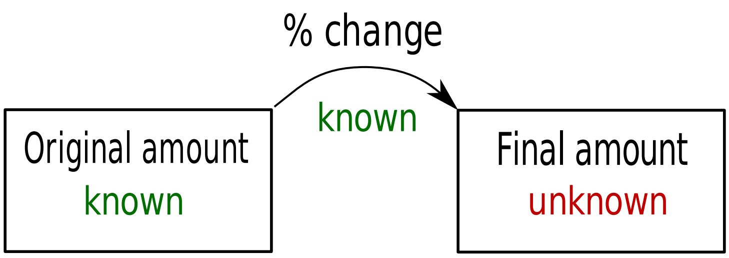
$$\begin{aligned}\text{Sale price} &= 0.87 \times 40 \\ &= £34.80\end{aligned}$$

Coumpound percentages

Even examples were percentages are added or deducted repeatedly can be performed in one step. If we return to the previous example of bank interest from the year 7 section: E.g. A man puts £400 into a savings account that pays 3% interest each year. How much will he have after 2 years?

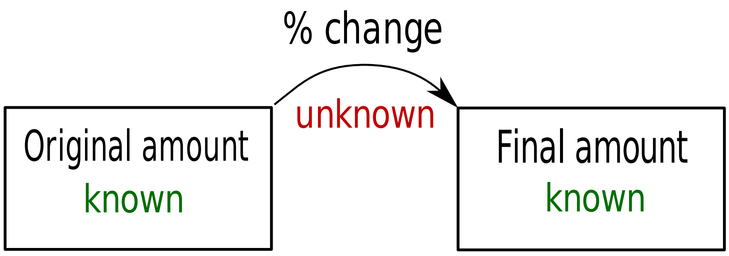
$$\begin{aligned}\text{New savings} &= (400 \times 1.03) \times 1.03 \\ &= 400 \times 1.032 \\ &= £424.36\end{aligned}$$

Since we are about to see other types of percentage calculation, it is worth summing up our previous percentage calculations using these "boxes" where we know the original amount and the change but wish to find the final amount:



3 Finding the percentage change (year 8 & 9)

Now take the situation where we know the original amount and the final amount but want to find the percentage change:



The following simple example can help us to derive a formula for this type of calculation. Imagine you got 10 commendations one week and 15 the next – it is clear to see that your commendations have increased by half, that is by 50%. This is because

$$\frac{5}{10} \times 100 = \frac{1}{2} \times 100 = 50\%.$$

Hence, we need to do:

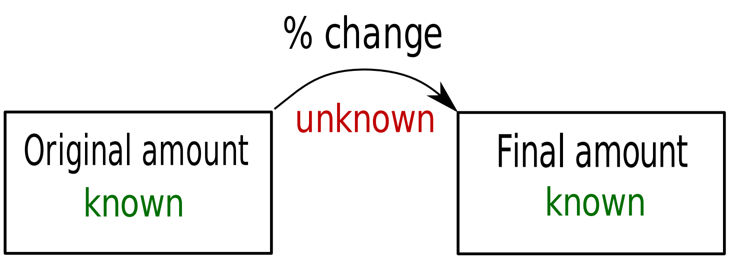
$$\text{Percentage change} = \frac{\text{actual change}}{\text{original value}} \times 100$$

Example. A car is bought for £12,000 and sold on for £7,500 a few years later. What is the percentage reduction in price? (this is called *depreciation*):

$$\begin{aligned}\text{Percentage decrease} &= \frac{\text{actual change}}{\text{original value}} \times 100\\ &= \frac{12,000 - 7,500}{12,000} \times 100\\ &= \frac{4,500}{12,000} \times 100\\ &= 37.5\%\end{aligned}$$

4 Reverse Percentages (years 8 & 9)

Now take the situation where we know the percentage change and the final amount, but not the original amount:



A very obvious error is seen below:

> A skirt costs £20 in a 10% sale, what was it full price? 10% of $20 = 2$ so full price is £22.
>
> But if we check this, 10% of £22 is £2.20 so the sale price would be £19.80. That is, we cannot find the percentage of the final amount since it is always the percentage of the original amount that we are working with!

The best approach is to use decimal multipliers and to try and reverse what we did in our previous calculations. Taking the above example:

$$\begin{aligned} \text{Full price of skirt} \times 0.9 &= \text{Sale price} \\ \text{Full price of skirt} &= \text{Sale price} \div 0.9 \\ &= 20 \div 0.9 \\ &= 22.2222222\ldots \\ &= \pounds 22.22 \end{aligned}$$

Example. What was my previous salary if I now receive £36,480 after a 14% pay rise?

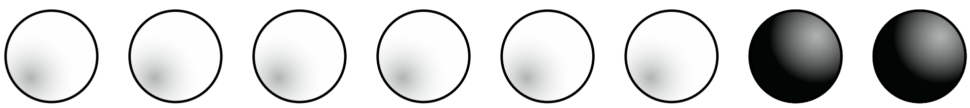
$$\begin{aligned} \text{Previous salary} \times 1.14 &= \text{New salary} \\ \text{Previous salary} &= \text{New salary} \div 1.14 \\ &= 36,480 \div 1.14 \\ &= 32,000 \\ &= \pounds 32,000 \end{aligned}$$

Example. What was my original weight if I know weigh 9 stone having lost 5% of my weight after having flu?

$$\begin{aligned} \text{Original weight} \times 0.95 &= \text{New weight} \\ \text{Original weight} &= \text{New weight} \div 0.95 \\ &= 9 \div 0.95 \\ &= 9.473684211\ldots \\ &= 9.47 \text{ stones (to 3s.f.)} \end{aligned}$$

5 One number as a percentage of another

Imagine that a bag contains 6 white beads and 2 black beads:



What percentage of these beads are black?

How many are black?	2
What fraction are black?	$\frac{2}{8}$ or $\frac{1}{4}$
What percentage are black?	$\frac{1}{4} \times 100 = 25\%$

So, it is best to think:

$$\text{How many?} \longrightarrow \text{What fraction?} \longrightarrow \text{What percentage?}$$

Example. A coffee shop sell coffee at £1.45 a cup, but keep 85p of this as profit. What is their percentage profit per cup of coffee that they sell?

$$\text{How many? } 85 \longrightarrow \text{What fraction? } \frac{85}{145} \longrightarrow \text{What percentage? } \frac{85}{145} \times 100$$

Therefore:

$$\begin{aligned} \text{Percentage profit} &= \frac{85}{145} \times 100 \\ &= 58.62068966\ldots \\ &= 58.6\% \text{ (to 1 d.p.)} \end{aligned}$$