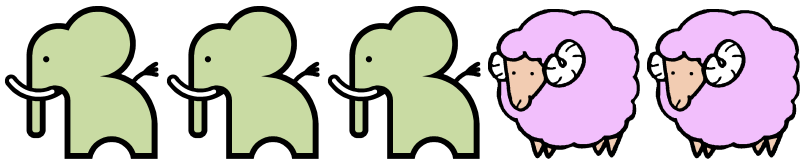
Ratio (7–9)

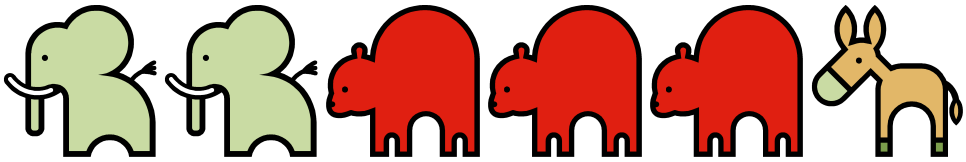
Contents

Introduction

A ratio is a way of comparing two or more quantities. For example:



The ratio of elephants to sheep is $3 : 2$.



The ratio of elephants to bears to horses is $2 : 3 : 1$.
The order of a ratio is important.

1 Simplifying ratios

If there are 20 boys and 10 girls in a class, the ratio of girls to boys is $10 : 20$. This can be simplified, by dividing both sides of the ratio by 10.

$$10 : 20$$
$$1 : 2$$

Example. Simplify each of the following:

$$4 : 6 = 2 : 3$$
$$0.5 : 7 = 1 : 14 \qquad \textit{(always write ratios with whole numbers)}$$
$$6 : 18 : 12 = 1 : 3 : 2$$

2 Working with ratios

Since we have seen that we can divide both sides of a ratio by an amount, it follows that we can multiply both sides by the same amount too.

Example. Imagine the ratio of shirts to ties in a wardrobe is $5 : 2$. If there are 15 shirts, how many ties are there?
(This means, for every 5 shirts you have 2 ties).

$$\begin{array}{c} 5 : 2 \\ {\scriptstyle \times 3}\downarrow \ \downarrow {\scriptstyle \times 3} \\ 15 : 6 \end{array}$$

So, there are 6 ties.

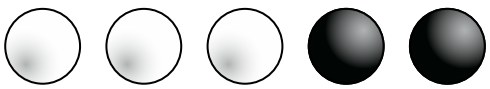
Example. Imagine the ratio of red beads to blue beads on a necklace is $4 : 1$ and the necklace has 30 beads in total. How many red beads are there?
This time, we have to work with the total:

$$\begin{array}{c} 4 : 1 \\ {\scriptstyle \times 6}\downarrow \ \downarrow {\scriptstyle \times 6} \\ 24 : 6 \end{array} \qquad \begin{array}{c} Total = 5 \\ \downarrow {\scriptstyle \times 6} \\ Total = 30 \end{array}$$

So, there are 24 beads.

3 The link to fractions

If the ratio of black beads to white beads in a bag is $2 : 3$, there could be 2 black and 3 white beads



The diagram shows that $\frac{2}{5}$ of the beads are black and $\frac{3}{5}$ are white.
So, a ratio of $4 : 9$ means that $\frac{4}{13}$ are on one quantity and $\frac{9}{13}$ the other.

4 Sharing in a given ratio

Two friends win £120 on the lottery and share it out in the ratio 1:5. How much does each friend get?
The ratio means that every time one friend gets £1, the other gets £5. If we use the link to fractions, one friend gets $\frac{1}{6}$ of the money and the other $\frac{5}{6}$.
As $\frac{1}{6}$ of the money is$\pounds 120 \div 6 = \pounds 20$:

$$\text{One friend gets } 1 \times \pounds 20 = \pounds 20$$
$$\text{The second friend gets } 5 \times \pounds 20 = \pounds 100$$

Example. Share 360g in the ratio $2 : 3 : 4$. Since we need to find $\frac{2}{9}$, $\frac{3}{9}$ and $\frac{4}{9}$, we start by finding the mass of $\frac{1}{9}$: $360 \div 9 = 40$. Then

$$2 \times 40 = 80g$$
$$3 \times 40 = 120g$$
$$4 \times 40 = 160g$$

Hence, 360g is shared into 80g, 120g and 160g.

5 Increasing or decreasing in a given ratio

Suppose you are holding a dinner party for 7 people and you use a recipe from a book which only serves 4. You have to adapt the amount of food for 4 people so that it is enough to serve 7. In this situation, we are increasing in the ratio $7 : 4$.

Example. Increase £300 in the ratio $5 : 2$. This is like saying that we have enough money for 2 people, how much would 5 people need if they all had the same amount?

$$\text{Each person} = \pounds 300 \div 2 = \pounds 150$$
$$\text{Five people} = \pounds 150 \times 5 = \pounds 750$$

Hence, £300 increased in the ratio $5 : 2$ is £750.

Example. Decrease 60g in the ratio $7 : 12$. This is like saying that we have enough to feed 12 people but we only need enough to feed 7.

$$\text{Each person} = 60 \div 12 = 5\text{g}$$
$$\text{Seven people} = 5 \div 7 = 35\text{g}$$

Hence, 60g decreased in the ratio $7 : 12$ is 35g.