

# Errors (9)

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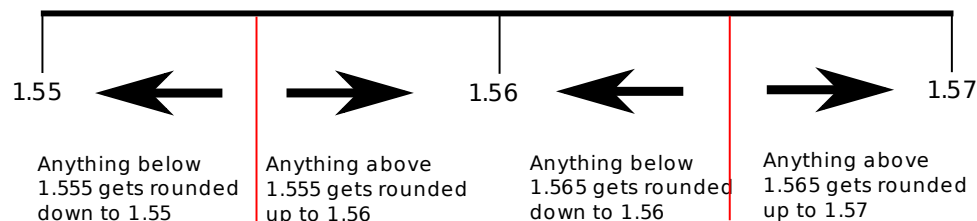
## Introduction

Whenever we take a measurement, it will not be totally accurate since it will rely on the accuracy of the measuring equipment that we are using.

If you measure your height to be 1.56metres, it may be a little more or a little less than this had the ruler had more intervals on it. For instance, you could have measured 1.56223897835..., but nobody really measures that accurately.

### 1 Upper and lower bounds

If you measured your height to be 1.56m, what is the smallest and the tallest that you could have measured? Consider the ruler that you may have used:



In between the two red lines, all values get rounded to 1.56 (up from 1.555 upwards and down from 1.565 downwards).

Hence, the least that you may have measured is 1.555...the is called the *lower bound*. The most that you may have measured is 1.565...this is called the *upper bound*.

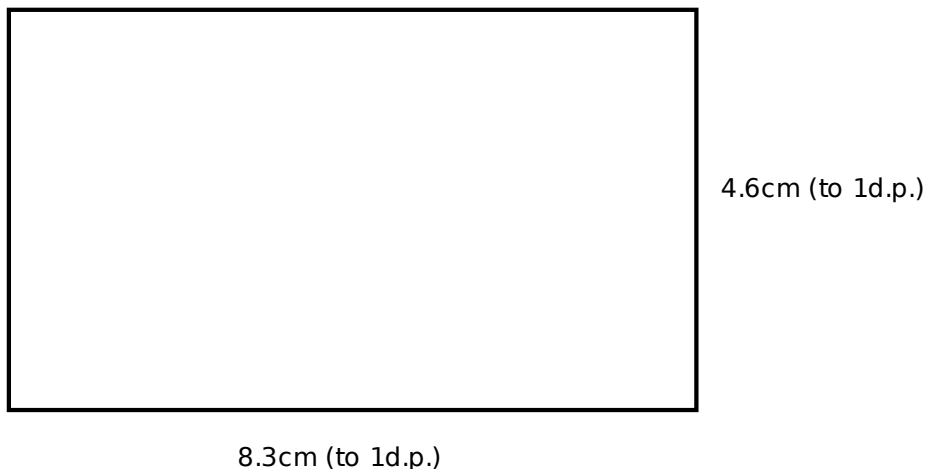
**N.B.** If you were 1.565m, this would actually get rounded up to 1.57. However, it would be a nuisance to write 1.5649999999999... so we take the *boundary* between one group and the next.

Look at this table of lower and upper bounds to get the idea.

Measurement	Lower bound	Upper bound
50 cm (to nearest cm)	49.5 cm	50.5 cm
50 cm (to nearest ten cm)	45 cm	55 cm
3.6 kg (to 1d.p.)	3.55 kg	3.65 kg
3.47 km (to 3s.f.)	3.465 km	3.475 km
65.0 secs (to 1d.p.)	64.95 secs	65.05 secs

## 2 Using errors in calculations

If a rectangle has measurements as shown, either one may have been measured inaccurately.



This means that if we use these measurements in a calculation, there is a range of answers that this solution could have taken. What is the range in possible areas of the above rectangle?

Smallest = lower bound $\times$ lower bound	Largest = upper bound $\times$ upper bound
$= 4.55 \times 8.25$	$= 4.65 \times 8.35$
$= 37.5375\text{cm}^2$	$= 38.8275\text{cm}^2$

The area can range from  $37.5375$  to  $38.8275\text{cm}^2$ .

Dont always combine lower bounds to get the lowest answer and upper bounds to get the largest. For instance, if  $a = 2.5$ (to 1dp) and  $b = 8.36$ (to 2dp), what is the smallest and largest value of  $\frac{b}{a}$ ?

Smallest = small no. $\div$ large no.	Largest = large no. $\div$ small no.
$= 8.3552.55$	$= 8.3652.45$
$= 3.276470588 \dots$	$= 3.414285714 \dots$

Therefore  $\frac{b}{a}$  ranges between  $3.276\dots$  and  $3.414\dots$