Significant figures & estimation (8 & 9)

Contents

Introduction

We have already seen that we can round numbers to a certain amount of decimal places. E.g.

$$\begin{aligned} 4.37995 &= 4.37|995 \\ &= 4.38 \text{ (to 2 d.p.)} \end{aligned}$$

We can also round numbers to a given amount of *significant figures*.

1 What are significant figures?

As a rule of thumb, significant figures are all of the digits in a number except for zeros at the beginning and at the end.

Number	Number of significant figures	Comment
36,000	2 s.f.	Dont count the three zeros at the end
5056	4 s.f.	We do count the zero in the middle
0.00089	2 s.f.	Dont count the four zeros at the beginning
34,567	5 s.f.	All the digits are significant

2 Rounding to a certain amount of significant figures

This is very similar to rounding decimal places. Count the number of significant figures that you need. Look to the next number: if it is less than 5, leave the number as it is but if it is more than five round up.

Example. Round 56,789 to 2 s.f.

$$\begin{aligned} 56,789 &= 56|789 && \textit{Next digit is } 7 \geq 5\ldots \\ &= 57,000 \text{ (to 2 s.f.)} && \ldots \textit{so round UP} \end{aligned}$$

(Notice how we fill in zeros in the empty spaces. You wouldnt like to wn £56,789 on the lottery and have it rounded to £57! It is close to £57,000)

Example. Round 0.00034 to 1s.f.

$$\begin{aligned} 0.00034 &= 0.0003|4 && \textit{Ignore leading zeroes} \\ &= 0.0003 \text{ (to 1s.f.)} \end{aligned}$$

Example. Round 45.6278 to 3s.f.

$$\begin{aligned} 45.6278 &= 45.6|278 \\ &= 45.6 \text{ (to 3s.f.)} \end{aligned}$$

There is one unusual case to consider. Round 0.999 to 1s.f.

$$\begin{aligned} 0.999 &= 0.9|99 && \textit{We need to add 1 to the 9 but we can't write 10 in a column} \\ &= 1 \text{ (to 1 s.f.)} && \textit{so we have to carry this along} \end{aligned}$$

Example. Round 3.987 to 2s.f.

$$\begin{aligned} 3.987 &= 3.9|87 \\ &= 4.0 \end{aligned}$$

It is important to keep the 0 as we need *two* significant figures.

3 Estimation

We can quickly and easily work out the answer to any calculation by performing an estimate. We should always do this to check that our answer is reasonable.
We dont want an estimate to take a long time (otherwise, we may as well do the full calculation), so the quickest idea is to round all numbers off to **1 significant figure**.

When estimating round all numbers to 1 s.f.

Example. Estimate the answer to $3.6 \times 10.9 + 194$.

$$\begin{aligned} 3.|6 &= 4 \text{ (to 1s.f.)} \\ 1|0.9 &= 10 \text{ (to 1s.f.)} && \textit{(again, we need to add a final zero)} \\ 1|94 &= 200 \text{ (to 1s.f.)} \end{aligned}$$

So, a good estimate is $4 \times 10 + 200 = 240$.

Example. Estimate the answer to $4.2 + 9.8 \times 19.4$.

$$\begin{aligned} 4.2 + 9.8 \times 19.4 &\approx 4 + 10 \times 20 && \textit{(Remember BODMAS)} \\ &\approx 4 + 200 \\ &\approx 204 \end{aligned}$$

Example. Estimate $\dfrac{9.71-3.89}{0.47}$.

$$\begin{aligned}\frac{9.71-3.89}{0.47} &\approx \frac{10-4}{0.5}\\ &\approx \frac{6}{0.5}\\ &\approx 12\end{aligned}$$

It would be tempting to think the answer is 3 but 0.5 fits 12 times into 6.