Surface area & volume (7–9)

Contents

Introduction

We need to understand the two terms *surface area* and *volume* and ensure that we do not confuse them.

Let us think about the difference first of all. Imagine various bars of chocolate:

Surface area — the wrapping or packaging.

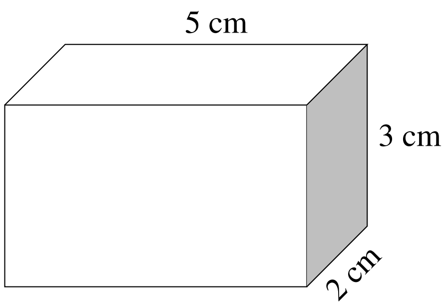
Volume — the chocolate.

That is, surface area is the "outer layer" of a solid, whereas volume is the "inside".

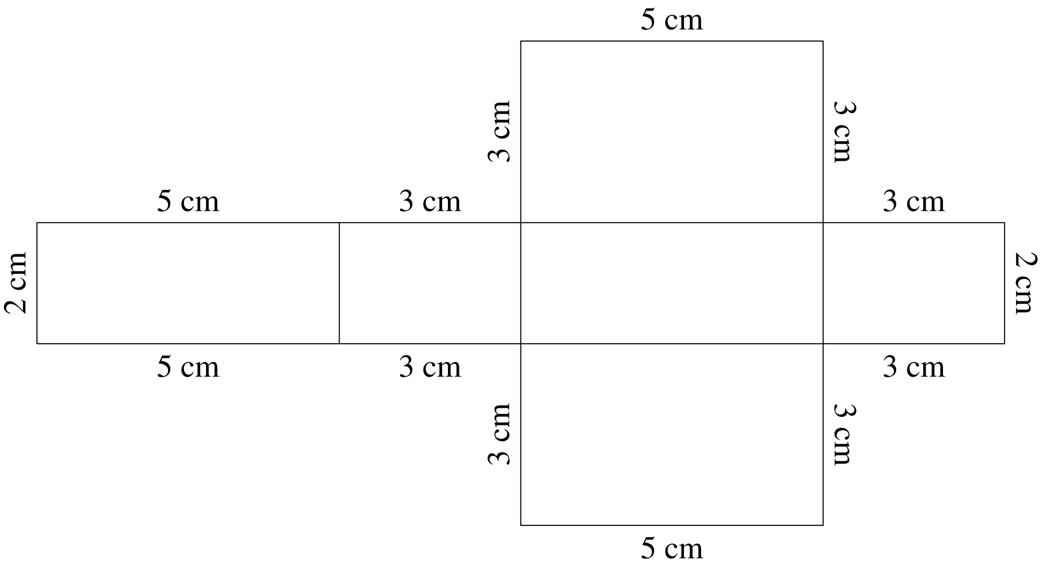
1 Surface area (7–9)

Like area, this is measured in square units such as cm^2 or m^2. Surface area is the total area of all of the faces of a solid added together (there are some special formulae e.g. for sphere, but these are needed only at GCSE level).

Example. Find the surface area of the following cuboid:



You may find that imagining the net would be a helpful step to finding the surface area:



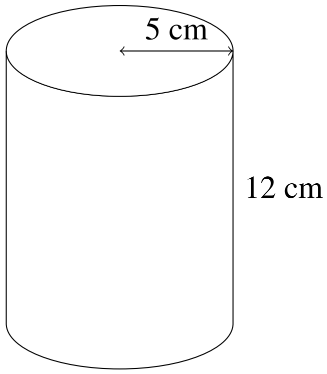
We have to find area of six rectangles, but we notice there are two of each type:

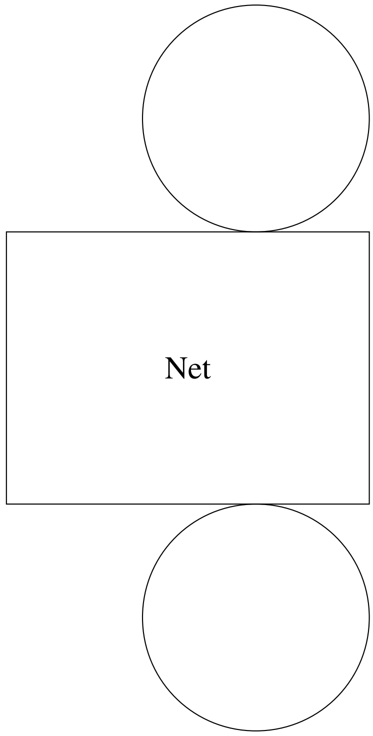
$$\begin{aligned}\text{First rectangles area} &= lw \\ &= 3 \times 5 \\ &= 15 \text{ cm}^2 \\ \text{Second rectangles area} &= lw \\ &= 2 \times 3 \\ &= 6cm2 \\ \text{Third rectangles area} &= lw \\ &= 2 \times 5 \\ &= 10 \text{ cm}^2\end{aligned}$$

So,

$$\begin{aligned}\text{Total surface area} &= (2 \times 15) + (2 \times 6) + (2 \times 10) \\ &= 62 \text{ cm}^2\end{aligned}$$

Example. (year 8+) Find the surface area of this cylinder:

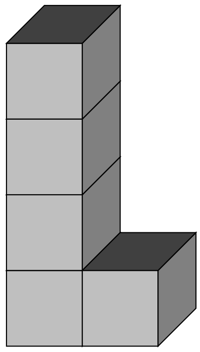




$$\begin{aligned}
\text{Each circle's area} &= \pi r^2 \\
&= \pi \times 5^2 \\
&= 25\pi \\
\text{Rectangle area} &= lw \\
&= 12 \times (2\pi \times 5) \\
&= 120\pi \\
\text{Total surface area} &= 25\pi + 120\pi \\
&= 145\pi \text{ cm}^2 \\
&= 455.5309348\ldots \\
&= 456 \text{ cm}^2 \text{ (to 3 s.f.)}
\end{aligned}$$

The rectangle's length wraps around the circle when rolled into a cylinder and so is equal to the circumference

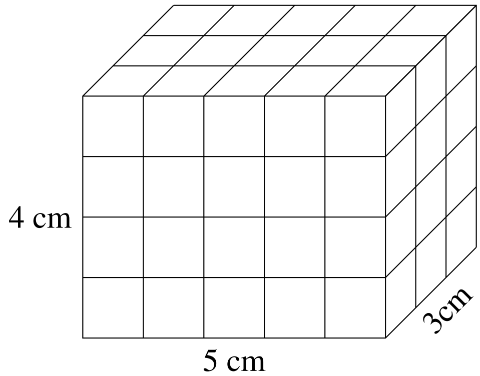
2 Volume (years 7–9)



This is the amount that we can fit "inside" a solid. The best way to measure the inside is to fill a shape with cubes. Hence, we measure volume in centimetre cubes (cm^3), metre cubes (m^3) etc. The solid to the right has a volume of 5 cm^3 since it contains 5 cubes.

2.1 Volume of a cube or cuboid (7)

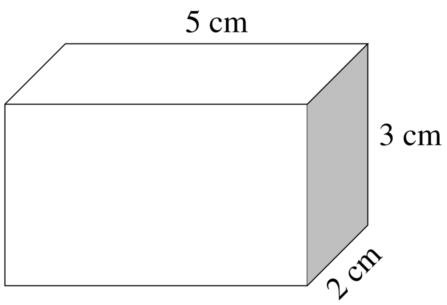
Imagine filling a 3 cm by 4 cm by 5 cm cuboid with little cm cubes (cm^3):



The front "layer" has 20 cubes in it and we can see, working backwards, 3 of these layers. Hence, the volume is 60 cm^3. Rather than counting the cubes, we notice that we could have simply done $3 \times 4 \times 5$. Hence:

$$\text{Volume of a cube or cuboid} = \text{length} \times \text{width} \times \text{height}$$
$$V = lwh$$

Example. Find the surface area of the following cuboid:



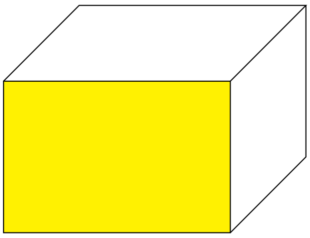
$$\begin{aligned} V &= lwh \\ &= 2 \times 3 \times 5 \\ &= 30 \text{ cm}^3 \end{aligned}$$

Example. What is the length of each edge of a cube with volume 125 m^3?

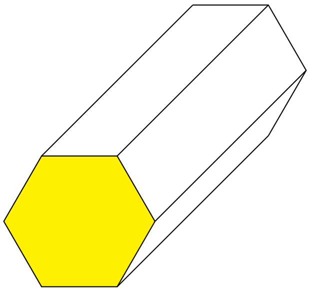
$$\begin{aligned} V &= lwh \\ V &= l \times l \times l && \textit{since all the lengths are equal} \\ V &= l^3 \\ 125 &= l^3 && \textit{Substitute the known value} \\ l &= \sqrt[3]{125} \\ l &= 5 \text{ m} \end{aligned}$$

2.2 Volume of prisms (8 & 9)

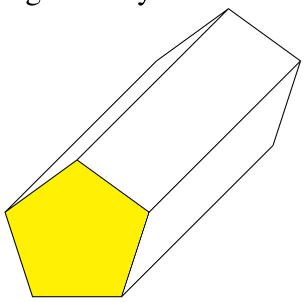
A prism is a solid that is the same shape all the way through. That is, it has constant cross-sectional area. Here are some prisms, with the *cross section* of each hilighted in yellow:



Rectangular prism



Hexagonal prism



Pentagonal prism

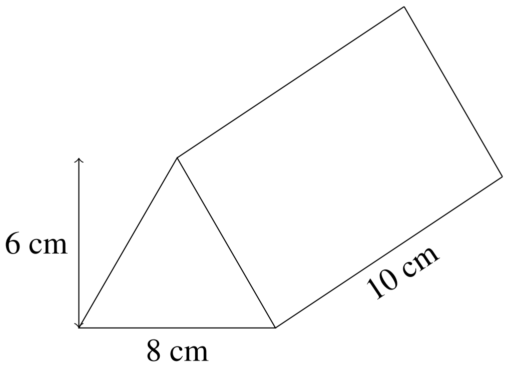
There are many prisms in real life, such as:

The “My maths” website gives a good explanation about how to find the volume of any prism. You need to find the area of the cross-section (so this would be a triangle in a toblerone or a circle in a tin of beans) and then multiply it by the height of the prism.

$$\begin{aligned} \text{Volume of prism} &= \text{Area of cross-section} \times \text{Height of Prism} \\ V &= Ah \end{aligned}$$

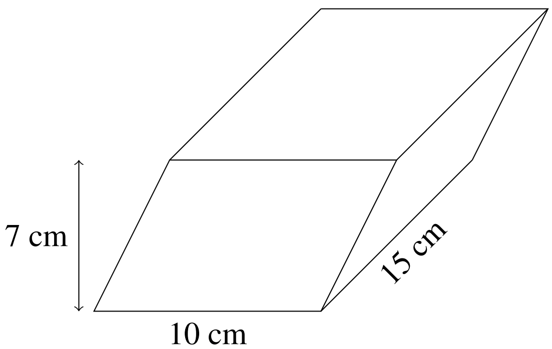
N.B. The term “height” can be a little misleading imagine your own height. When you lie down in bed at night your height becomes horizontal. Have a look at the “height” marked on these two prisms:

Example. Find the volume of the following prisms:



(the cross-section is a triangle)

$$\text{Area of cross-section} = \frac{1}{2}bh$$
$$A = \frac{1}{2} \times 8 \times 6$$
$$A = 12 \text{ cm}^2$$
$$\text{Volume} = Ah$$
$$V = 12 \times 10$$
$$V = 120 \text{ cm}^3$$



(the cross-section is a parallelogram)

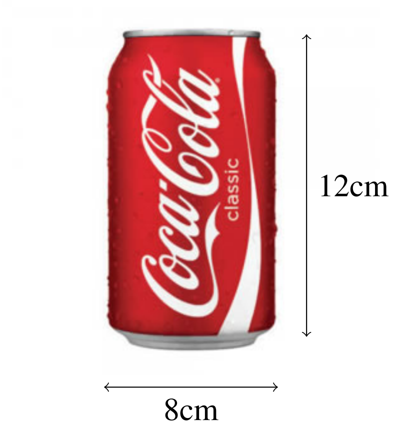
$$\text{Area of cross-section} = bh$$
$$A = 10 \times 7$$
$$A = 70 \text{ cm}^2$$
$$\text{Volume} = Ah$$
$$V = 70 \times 15$$
$$V = 1050 \text{ cm}^3$$

2.3 Volume of a cylinder (year 8 & 9)

A cylinder is also a prism, so we find its volume using $V = Ah$. However, the area of a circle is πr^2, so it is possible to remember the volume of a cylinder formula as:

$$V = \pi r^2 h, \qquad \text{where } r = \text{radius and } h = \text{height}$$

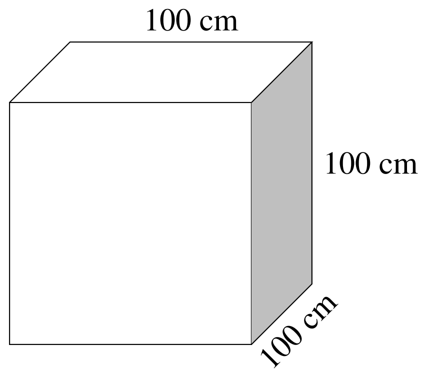
Example. Find the volume of this can:



$$
\begin{aligned}
V &= \pi r^2 h \\
&= \pi \times 4^2 \times 12 \\
&= 192\pi \\
&= 803.18578\ldots \\
&= 803 \text{ cm}^3 \text{ (to 3 s.f.)}
\end{aligned}
$$

2.4 A note on volume units

It may be tempting to think that there are, say, 100 cm^3 in 1 m^3. This is not the case. If you imagined a metre cube:



If we were to fill this cube with centimetre cubes, we would be able to fit 100 across the length, 100 across the width and 100 up the height. That is, we could fit

$$100 \times 100 \times 100 = 1,000,000 \text{ cm}^3$$

in one metre cube. So,

$$1 \text{ m}^3 = 1,000,000 \text{ cm}^3.$$

To remember the conversions, we need to remember that we are dealing with cubes and so *cube* our normal conversions. E.g.

$$
\begin{aligned}
1 \text{ cm}^3 &= 10 \times 10 \times 10 = 1,000 \text{ mm}^3 \\
1 \text{ m}^3 &= 100 \times 100 \times 100 = 1,000,000 \text{ cm}^3 \\
1 \text{ km}^3 &= 1,000 \times 1,000 \times 1,000 = 1,000,000,000 \text{ m}^3
\end{aligned}
$$