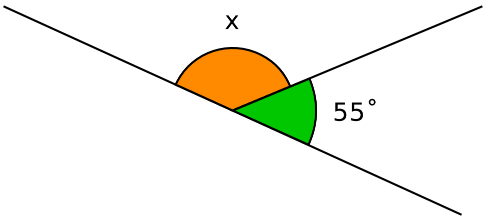
Rules of angles (7–9)

Contents

1 basic rules of angles

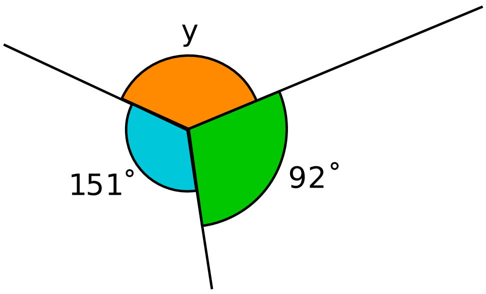
There are various *Rules of angles* that you should know. These can be used in any geometrical diagram to work out missing angles without the diagram having to be drawn to scale. We do not need a protractor since the rule will give us the exact answer. The basic rules you should know are:

Angles on a straight line add to $180°$



$$\begin{aligned} x + 55 &= 180 \qquad \textit{Angles on a straight line} \\ x &= 125° \end{aligned}$$

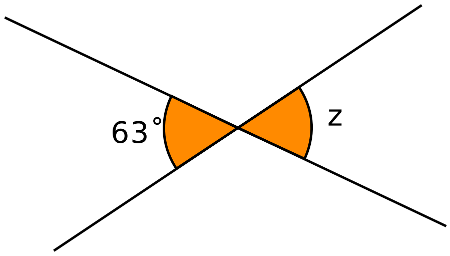
Angles at a point add to $360°$



$$\begin{aligned} y + 92 + 151 &= 360 \qquad \textit{Angles at a point} \\ y + 243 &= 360 \\ y &= 117° \end{aligned}$$

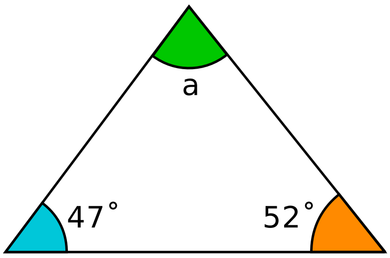
Vertically opposite angles are equal

Note: this is not like angles at a point since here we are dealing with where two straight lines intersect, like a pair of scissors:



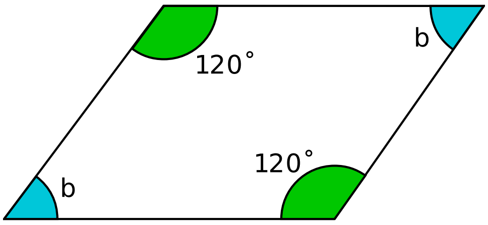
$z = 63°$ *Vertically opposite angles*

Angles in a triangle add to $180°$



$$\begin{aligned} a + 47 + 52 &= 180 \quad \textit{Angles in a triangle} \\ a + 99 &= 180 \\ a &= 81° \end{aligned}$$

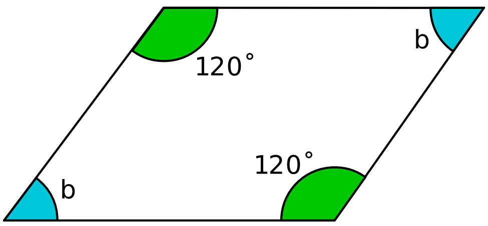
Angles in a quadrilateral add to $360°$



$$\begin{aligned} b + 120 + b + 120 &= 360 \quad \textit{Angles in a quadrilateral} \\ 2b + 240 &= 360 \\ 2b &= 120 \\ b &= 30° \end{aligned}$$

Notice how, in each case, we set out our working clearly using a logical algebraic layout and we always give the reason for a particular angle.

Example. Find x and y in the following diagram:



To find x:

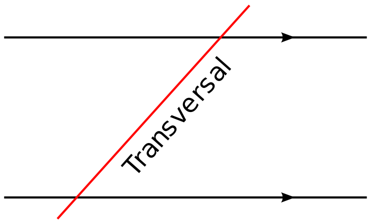
$$\begin{aligned} x + 75 &= 180 \quad \textit{Angles on a straight line} \\ x &= 105° \end{aligned}$$

To find y:

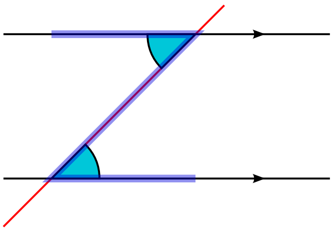
$y = 85°$ *Vertically opposite angles*

2 Angles in parallel lines (7–9)

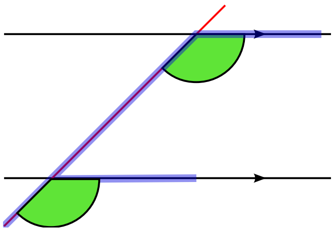
When a line passes through a pair of parallel lines, this line is called a transversal:



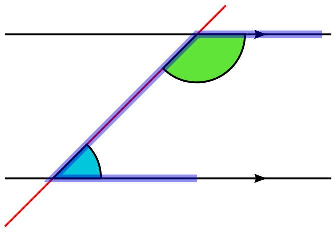
A transversal creates three letters of the alphabet which hide 3 new rules of angles:



Alternate angles
are equal
(Z-angles)

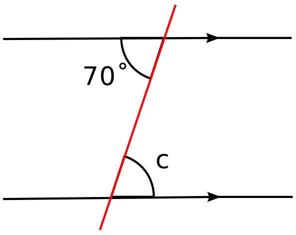


Corresponding angles
are equal
(F-angles)

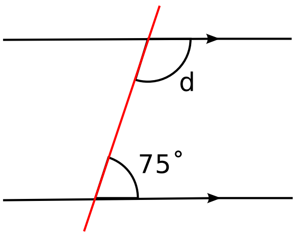


Interior angles
add to 180°
(C-angles)

Have a look at these examples:

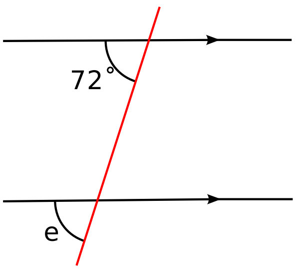


$c = 70°$ *Alternate angles*



$d + 75 = 180$ *Interior angles*

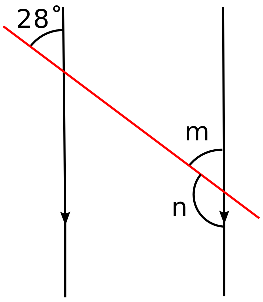
$d = 105°$



$e = 72°$ *Corresponding angles*

$d = 105°$

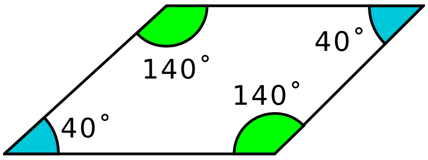
Note that the “F” is back to front!



$$m = 28^\circ \qquad \textit{Corresponding angles}$$
$$m + n = 180^\circ \qquad \textit{Angles on a straight line}$$
$$n = 152^\circ$$

Angles in quadrilaterals

We have already seen that the angles in any quadrilateral add up to 360°. There is an interesting special case that allows us to use what we have just learned about angles in parallel lines:



In a parallelogram, angles next to each other make a "C" shape (interior angles). This means that they add up to 180°. Therefore,

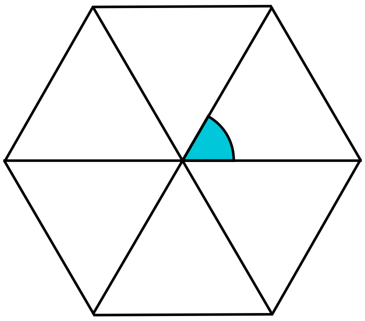
> In a parallelogram, opposite angles are equal.

3 Angles in polygons (year 9)

- A *polygon* is a shape with straight sides.
- A *regular polygon* has all sides and all angles equal.

We may need to find several angles in polygons.

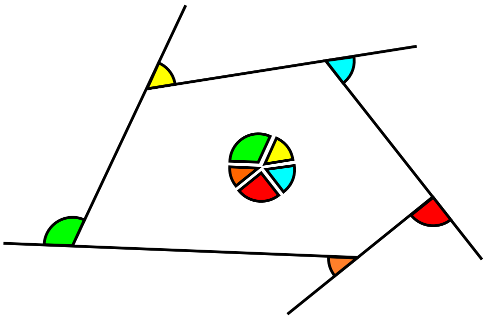
3.1 The central angle in a regular polygon



The angles sit around a circle and so add to 360°. Each angle is $360 \div n$, where n is the number of sides of the polygon.
E.g. here we have a hexagon:

$$\text{Each angle is } 360 \div 6 = 60^\circ$$

3.2 The exterior angle of any polygon



In any polygon, the exterior angles are found where the extension of a side meets the next side, as the diagram shows. Since these extensions all form a "windmill" effect, their total turn is equivalent to a full circle.

$$\text{Sum of exterior angles} = 360°$$

Example. What is the exterior angle of a regular pentagon?

Each angle is equal as the pentagon is regular. Therefore,

$$\begin{aligned} \text{Each angle} &= 360 \div 5 \\ &= 72° \end{aligned}$$

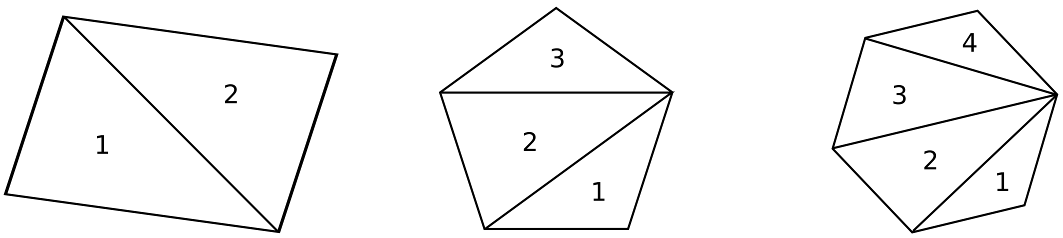
3.3 The interior angle of any polygon

We know that:

- in a triangle, interior angles add to $180°$;
- in a quadrilateral, interior angles add to $360°$.

If we follow the pattern, we notice that the total goes up by $180°$ each time.

But why is this? If we take one vertex of any polygon and join it to all of the others, we create triangles:



Quadrilateral 2 triangles: $2 \times 180 = 360°$

Pentagon 2 triangles: $3 \times 180 = 540°$

Hexagon 2 triangles: $4 \times 180 = 720°$

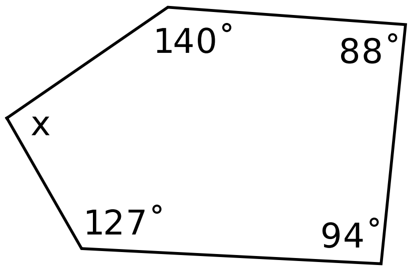
Notice also that the number of triangles needed is always two less than the number of sides in the polygon. So in general:

$$\left[\begin{array}{c}\text{Sum of} \\ \text{interior angles}\end{array}\right] = 180(n-2), \quad \text{where } n \text{ is the number of sides}$$

Moreover, if the polygon is regular, we can divide the sum by n to obtain the size of each interior angle. The following table sums these up for a few polygons:

Number of sides	n	3	4	5	6	7	8	9	10
Number of triangles	$n-2$	1	2	3	4	5	6	7	8
Sum of angles	$180(n-2)$	180	360	540	720	900	1080	1260	1440
Each angle if regular	$\frac{180(n-2)}{n}$	60	90	108	120	128.57	135	140	144

Example. What is the missing angle below?



In a pentagon, the sum of the interior angles is $540°$.

$$\begin{aligned} x + 135 + 130 + 75 + 120 &= 540 \\ x + 460 &= 540 \\ x &= 80° \end{aligned}$$

Example. What is the size of any interior angle in a regular dodecagon? (NB A dodecagon has 12 sides)

A 12 sided shape can be divided into 10 triangles.

$$\begin{aligned} \text{Sum of interior angles} &= 10 \times 180° \\ &= 1800° \end{aligned}$$

Therefore

$$\begin{aligned} \text{Each interior angle} &= 1800 \div 12 \\ &= 150° \end{aligned}$$