locus (8–9)

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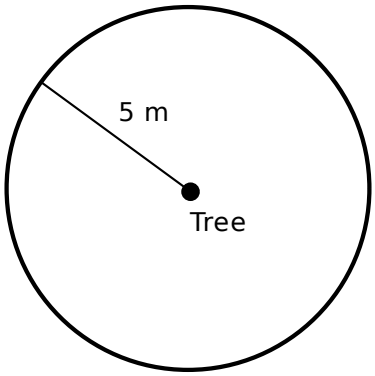
1 What is a locus?

A locus is the path of any object as it moves under certain conditions. For example you may draw a locus as if you are taking a bird's eye view of the situation.

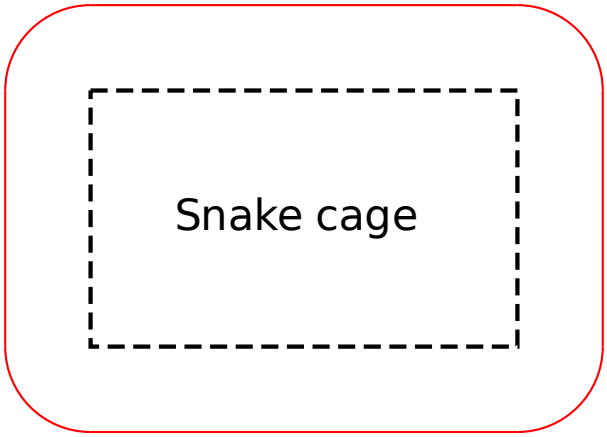
Example. A goat is tied to a 5m long rope which, in turn, is tied around a pole. If the rope is taut (that is, tight), show the locus of the goat.
NB You don't put lots of detail such as trees and goats on your diagram!



Locus of the goat

The goat would be able to walk around the circumference of a circle with radius 5m (the pole is at the centre of this circle).

Example. A snake is in a cage at a zoo. The cage measures 6m^2. Spectators are warned not to go over a red line, marked 2m from the cage. Show the locus of the red line.



Locus of the red line

This rough sketch shows 4 lines 2m from the cage, with rounded corners to keep each point 2m from any point on the cage.

2 Accurate constructions

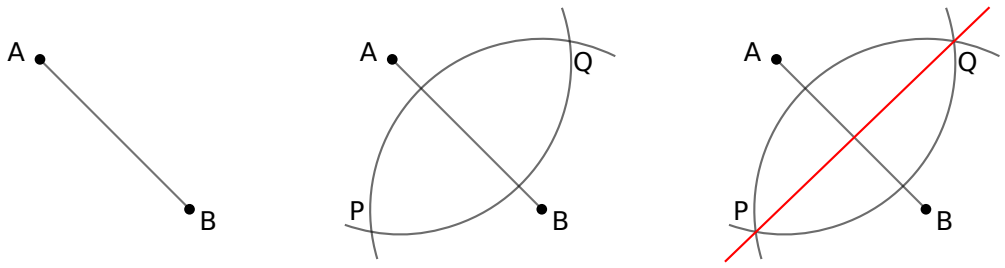
You will need to know how to perform certain constructions in order to construct loci accurately:

- A circle (like the goat example): make sure you can use a compass.
- A perpendicular bisector.
- An angle bisector.

2.1 Perpendicular bisector

This is the set of all points that are equidistant (an equal distance) from two fixed points.

1. Draw a line connecting your two points and open out your compass more than half way along this line
2. With your compass point on one of the two points, construct an arc crossing the line joining the two points Repeat for the other fixed point.
3. Where your two arcs intersect, join with a ruler. This final line is the perpendicular bisector and any point on it is exactly half way between the original two points drawn.



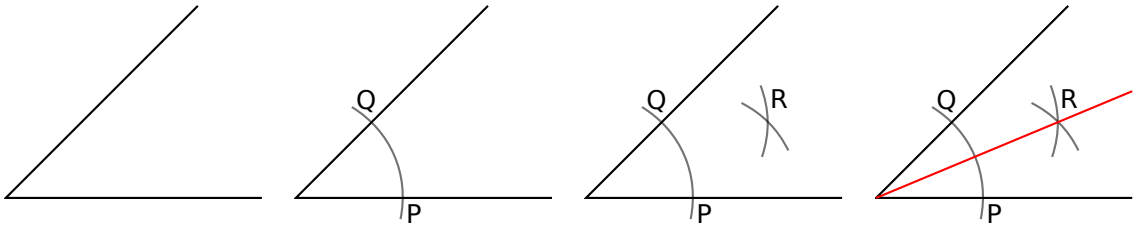
Step 1 *Step 2* *Step 3*

You can use the MyMaths website to see a perpendicular bisector being constructed from scratch.

2.2 Angle bisector

This is the set of all points that are equidistant (an equal distance) from two fixed lines.

1. Draw two fixed lines
2. Open out your compass and put the point on the intersection of the two lines, drawing a large arc crossing the two lines
3. Move your compass to the point where the arc intersects either line and draw another arc. Repeat for the other intersection.
4. Join the intersection of these two arcs to the intersection of the original two lines. This final line is the angle bisector and any point on it is exactly half way between any point on the two original lines.



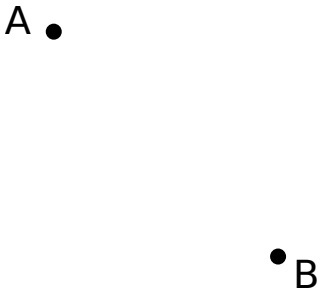
Step 1 *Step 2* *Step 3* *Step 4*

You can use the MyMaths website to see an angle bisector being constructed from scratch.

N.B. A perpendicular bisector is equidistant from two points, an angle bisector from two lines.

2.3 An example

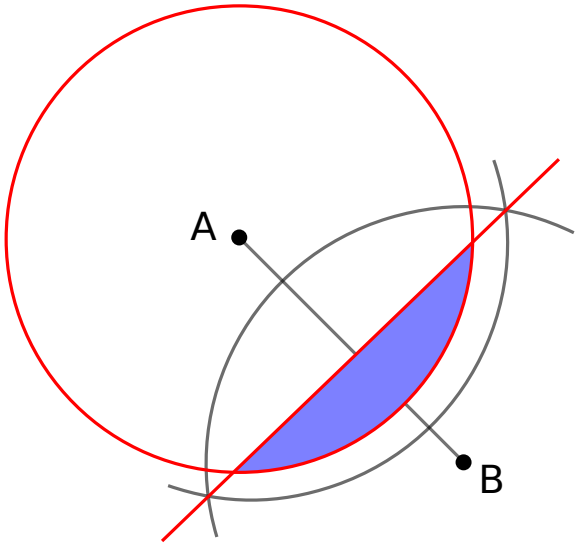
Two towns A and B are 8 miles apart. A phone mast is to be constructed closer to B than A but at most 6 miles from A. Show where the mast can go.



Clue 1. Don't worry about where closer to A than B is yet: just worry about where half way is first. Half way between A and B (two points) is the *perpendicular bisector*. We would then shade points on the right of this line (closer to B).

Clue 2. Don't worry about where less than 6 miles from A is yet: just worry about where 6 miles exactly is first. This is a *circle* with radius 6 miles and centred at A. We would then shade points inside this circle, but only those already covered by clue 1.

With these clues we can find the locus of the points where that mast can be:



The blue shaded area is where the mast can go.