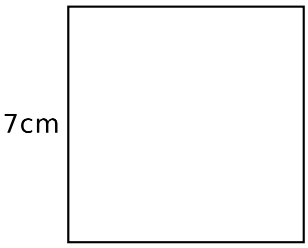
Perimeter and area, including circles (7–9)

Contents

1 Perimeter (year 7)

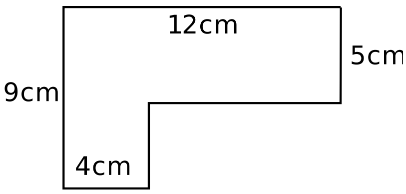
The *perimeter* is the total distance around the outside of a shape. To find the perimeter of a shape, simply add all of the lengths of the sides together (a circle is the special case and we will deal with this later).

Example. Find the perimeter of a square with side 7cm.



$$\text{Perimeter} = 7 + 7 + 7 + 7$$
$$= 49 \text{ cm}$$

Example. Find the perimeter of the following shape.



This shape is a hexagon (that is, it has 6 sides), so make sure that six lengths are added together. We have only been given four lengths so need to work out the two missing lengths:

$$\text{Missing horizontal length} = 124 = 8 \text{ cm}$$
$$\text{Missing vertical length} = 95 = 4 \text{ cm}$$
$$\text{So, Perimeter} = 9 + 12 + 5 + 8 + 4 + 4 \quad \textit{Go clockwise from bottom left corner}$$
$$= 42 \text{ cm}$$

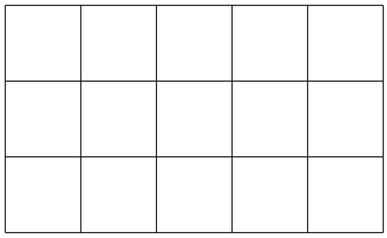
2 Area of polygons(year 7–9)

The *area* of a shape is the total surface covered by it. To cover a surface, the best way is to use lots of squares, so we measure areas in units squared. E.g. if a shape has an area of 10 cm^2, it means that its surface is the same as that of 10 squares of 1 cm by 1 cm.

For common shapes, we have special formulae to work out their areas.

2.1 Squares and rectangles (year 7)

Imagine a rectangle measuring 3cm by 5cm:



By counting squares we can see that the area is 15cm^2. However, we notice it is much quicker to do 3×5. So:

$$\text{Area of a square or rectangle} = \text{length} \times \text{width}$$
$$A = lw$$

Example. What is the area of a rectangle with width 2cm and length 8cm?

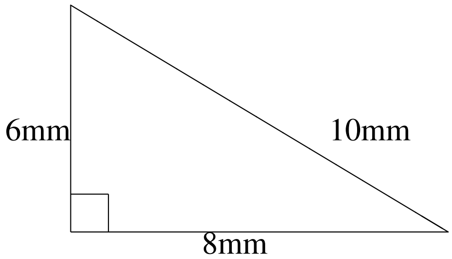
$$\begin{aligned} \text{Area} &= lw && \textit{Write the formula first} \\ &= 2 \times 8 && \textit{Subsitute in the numbers} \\ &= 16 \text{ cm}^2 && \textit{Include the units in your answer} \end{aligned}$$

Notice how our layout includes a formula, working, answer and units.
For the next formulae, use the MyMaths website to see how they have been derived. Learn all of these formulae off by heart.

2.2 Triangles (year 7)

$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$$
$$A = \frac{1}{2}bh$$

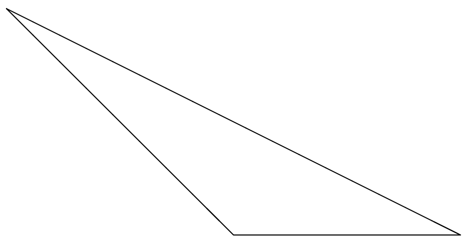
Example. Find the area of this triangle:



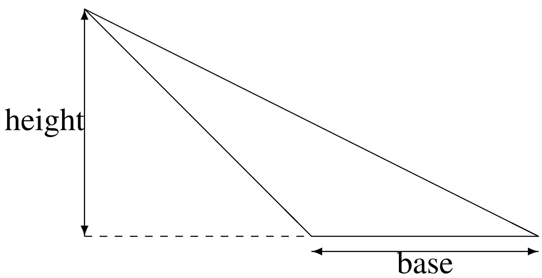
We need to decide which is the base and which is the perpendicular height. If we assume that the 8mm side is the base, we now want to choose the side at right angles to this to give the triangle's height. In this case, this is the 8mm side. This is just like measuring your own height you wouldnt stand at an angle against the wall, you would stand up straight, making a right-angle with the floor. So:

$$\begin{aligned} \text{Area} &= \frac{1}{2}bh && \textit{Write the formula first} \\ &= \frac{1}{2} \times 8 \times 6 && \textit{Then substitute in the value} \\ &= 24 \text{ mm}^2 && \textit{Include units in your answer} \end{aligned}$$

Question. If you were finding the area of the following triangle, where would the base and height be?

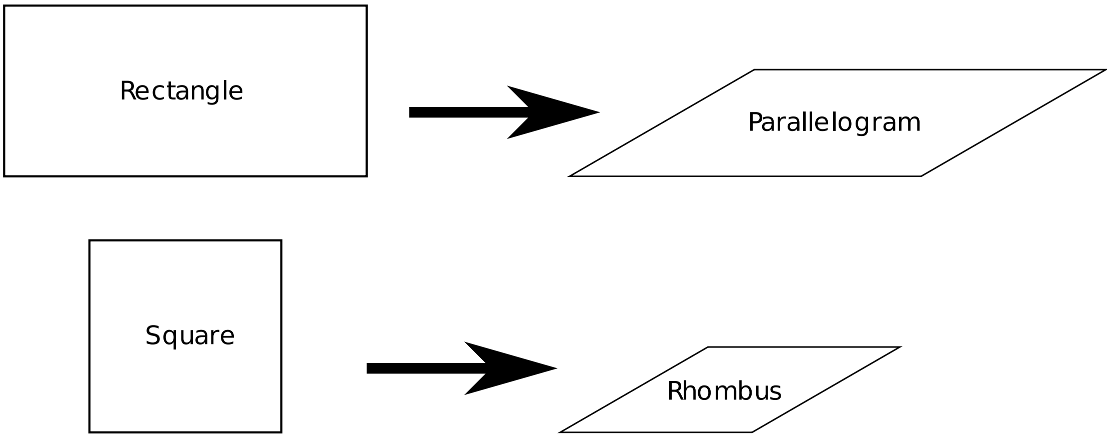


Answer. Dont forget to look for the "straight-up" height:



2.3 Area of a parallelogram or rhombus (year 8)

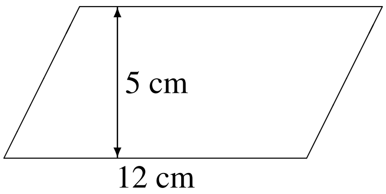
A parallelogram is a rectangle that has been given a "push" and a rhombus is a square that has been given a "push" (this is the shape that you might know as a diamond):



The formula for the area of a parallelogram or rhombus is the very similar to that for a rectangle or square (length $\times$ width):

$$\text{Area of parallelogram or rhombus} = \text{base} \times \text{perpendicular height}$$
$$A = bh$$

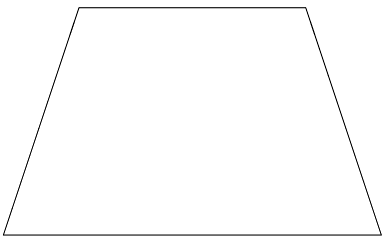
Example. Find the area of this parallelogram:



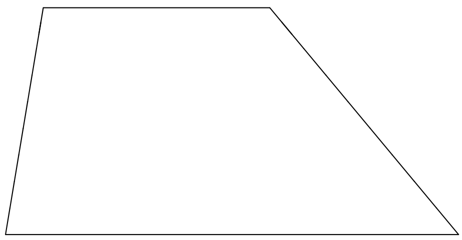
$$\begin{aligned}\text{Area} &= bh \\ &= 12 \times 5 \\ &= 60 \text{ cm}^2\end{aligned}$$

2.4 Area of a trapezium

A trapezium is a four-sided shape (quadrilateral) with one pair of parallel sides e.g.



Isoceles trapezium
Slanted sizes are equal in length



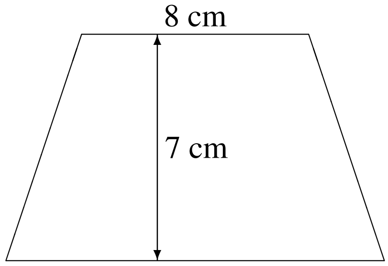
Trapezium
Slanted sizes are not equal

To find the area of a trapezium, we use:

$$\text{Area of trapezium} = \frac{1}{2} \times \begin{bmatrix}\text{Sum of} \\ \text{parallel sides}\end{bmatrix} \times \begin{bmatrix}\text{Perpendicular} \\ \text{height}\end{bmatrix}$$

$$\text{Area} = \frac{1}{2}(a+b)h$$

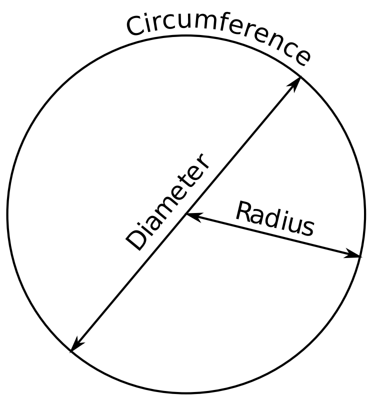
Example. Find the area of this shape:



$$\begin{aligned}\text{Area} &= \frac{1}{2}(a+b)h \\ &= \frac{1}{2} \times (12+8) \times 7 \\ &= \frac{1}{2} \times 20 \times 7 \\ &= 70 \text{ cm}^2\end{aligned}$$

3 Circles (year 8)

You need to be able to find both the perimeter and the area of a circle. To do this, you will need to know the vocabulary associated with a circle.



Radius: distance from the centre to the outside edge

Diameter: distance all the way across the circle, through the centre

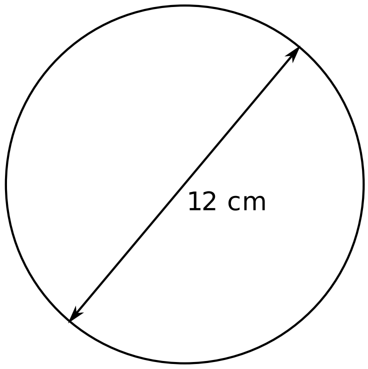
Circumference: the length of the outside edge

3.1 Perimeter of a circle

The perimeter of a circle is called the *circumference*.

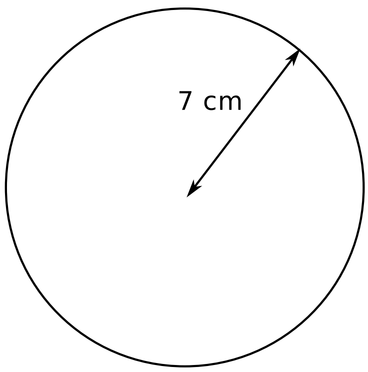
$$\text{Circumference} = \pi D \qquad \pi = 3.141592\ldots \text{ ("pi")}$$
$$D = \text{diameter of circle}$$

Example. Find the circumference of the following circle.



$$
\begin{aligned}
C &= \pi D \\
&= \pi \times 12 \\
&= 12\pi && \textit{You may have to leave it terms of } \pi \\
&= 37.69911184\ldots \\
&= 37.7 \text{ cm (3 s.f.)} && \textit{or give a numerical answer}
\end{aligned}
$$

Example. n this next example we have to double the radius to get a diameter. The radius is 7cm so the diameter will be 14cm.



$$
\begin{aligned}
C &= \pi D \\
&= \pi \times 14 \\
&= 14\pi && \textit{You may have to leave it terms of } \pi \\
&= 43.98229715\ldots \\
&= 44.0 \text{ cm (3 s.f.)} && \textit{or give a numerical answer}
\end{aligned}
$$

N.B. Since a diameter is the same as two radii, then the formula can be remembered in a different way:

$$
\begin{aligned}
C &= \pi D \\
C &= \pi x 2r \\
C &= 2\pi r
\end{aligned}
$$

3.2 Area of a circle

The area of any circle is found using the formula:

Area $= \pi r^2$	$\pi = 3.141592\ldots$ $r =$ radius

N.B. Thinking about BODMAS, we need to square the radius and then multiply by π to find the area.

Example. Find the area of the last cirle in the previous section:

$$\begin{aligned}\text{Area} &= \pi r^2 \\ &= \pi \times 7^2 && \textit{Remember to square first} \\ &= 49\pi \\ &= 153.93804\ldots \\ &= 154 \text{ cm}^2 \text{ (to 3sf)}\end{aligned}$$

Example. Find the area of a bicycle wheel which travels 150cm in one revolution.
The distance travelled in one turn (revolution) is the same as the circumference:

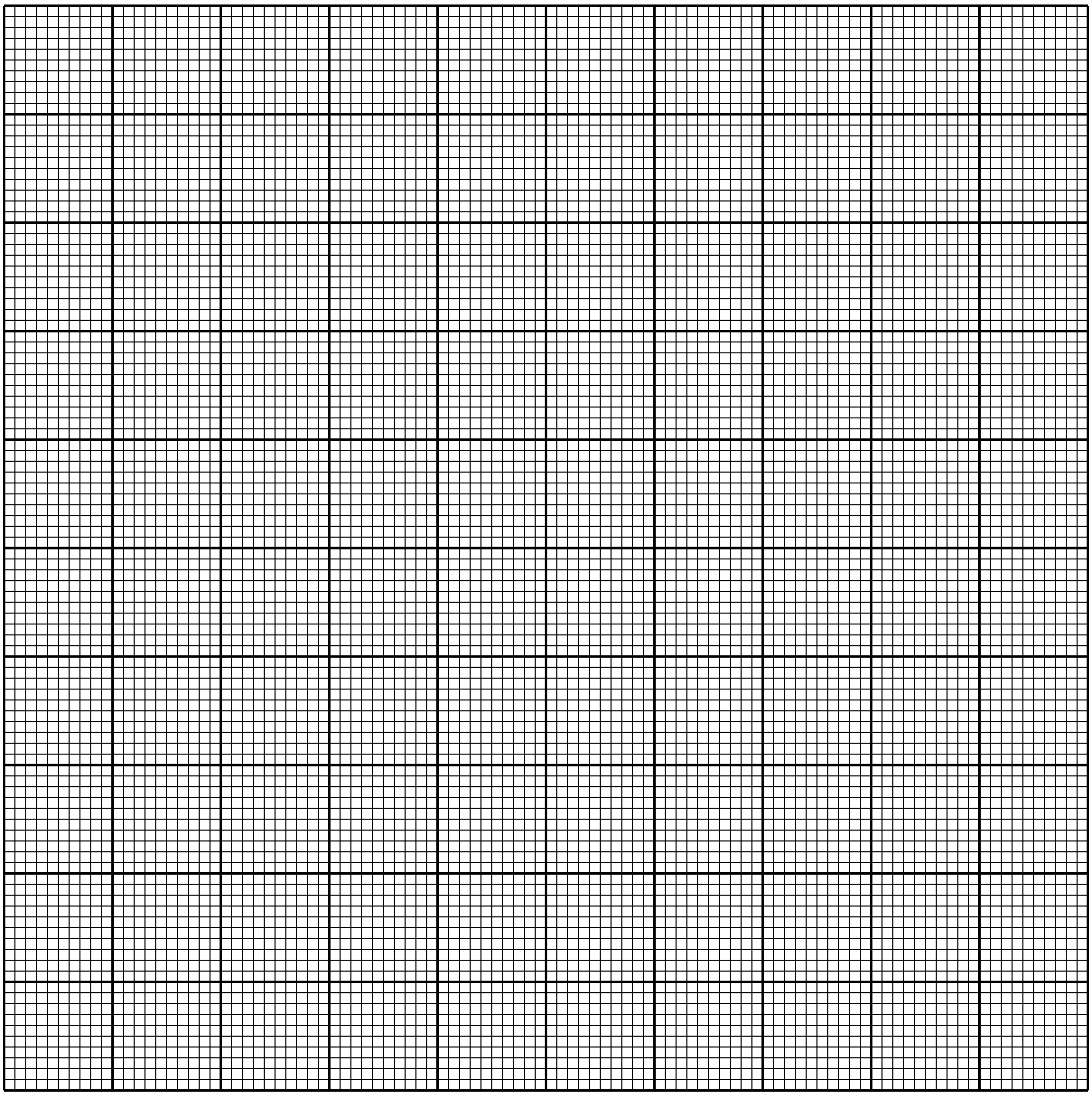
$$\begin{aligned}C &= \pi D \\ 150 &= \pi D && \textit{“I think of a number and multiply it by } \pi\textit{”} \\ D &= \frac{150}{\pi} && \textit{Don't work this out yet — we haven't finished the question}\end{aligned}$$

Since we know the diameter, we halve this to get the radius:

$$\begin{aligned}A &= \pi r^2 \\ A &= \pi \times \left(\frac{1}{2} \times \frac{150}{\pi}\right)^2 \\ A &= 1790.49311\ldots \\ A &= 1790 \text{ cm}^2 \text{ (to 3 s.f.)}\end{aligned}$$

4 A note on area units

You need to be very careful converting between units of area. For example, you might think that 200 cm^2 is equivalent to 2 m^2. This is not the case. Think about this diagram, imagining this is one metre by one metre:



Each little square is $1\ \text{cm}^2$ — there are 100 of theses across the legth and 100 across the height. This means that in total there are $100 \times 100 = 10,100\ \text{cm}^2$ in $1\ \text{m}^2$. You can remember it like this: as the units involve squaring, square the usual conversion to get the area conversion:

$$1\ \text{cm}^2 = 10\ \text{mm} \times 10\ \text{mm} = 100\ \text{mm}^2$$
$$1\ \text{m}^2 = 100\ \text{cm} \times 100\ \text{cm} = 10,000\ \text{cm}^2$$
$$1\ \text{km}^2 = 1000\ \text{m} \times 1000\ \text{m} = 1,000,000\ \text{m}^2$$