# Similarity (9)

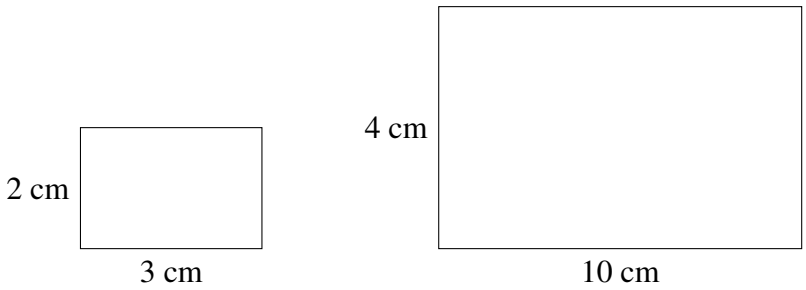
## Contents



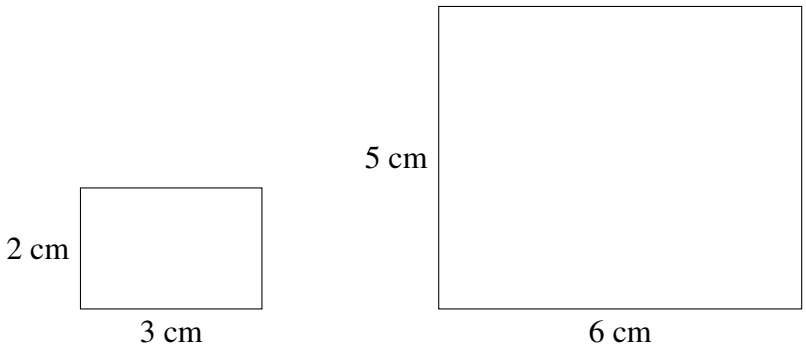
## 1 What is similarity?

Two shapes are *congruent* if they are exactly the same:

Two shapes are *similar* if one is an enlargement of the other:




Notice how both the length and width have doubled, producing similar rectangles. The following rectangles are not similar – each side has increased by 3 cm, but enlargement involves multiplication by a scale factor:



We notice, then, that similar shapes have sides in the same proportion. If we take the first pair of similar rectangles, we see that:

$$\frac{6}{3} = \frac{4}{2}$$

But, in the second pair that arent similar:

$$\frac{6}{3} \neq \frac{5}{2}$$

**Example.** Are these notes similar?

7 cm


12 cm

2.5 cm


4 cm

Does

$$\frac{12}{4} = \frac{7}{2.5} \quad ?$$

No, as $\frac{12}{4} = 4$ and $\frac{7}{2.5} = 2.8$. Therefore the sides are not in proportion.

## 1.1 Angles in similar figures

Look again at the first pair of similar rectangles are the start of these notes. We can see that each side has doubled in length but each rectangle still has four angles each of $90°$. This always works, even if the angles are not right angles:

| Similar figures have sides in the same proportion but their angles are the same. |
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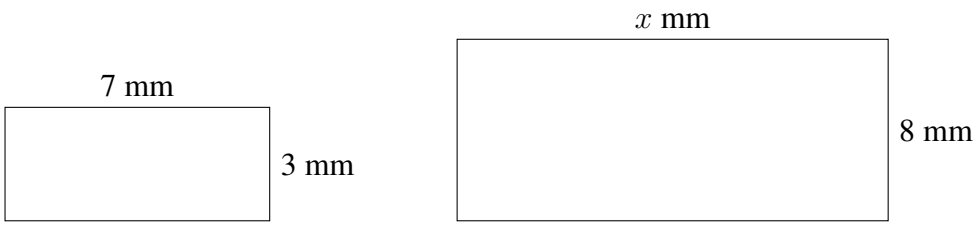
## 1.2 Problems with similar figures

Find the value of $x$ in the following diagrams.



Since the sides are in the same proportion then:

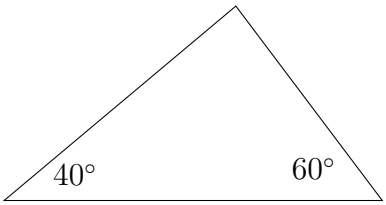
$$\frac{x}{7} = \frac{8}{3}$$
$$x = \frac{8}{3} \times 7$$
$$x = \frac{56}{3}$$
$$x = 18\frac{2}{3} \text{ mm}$$

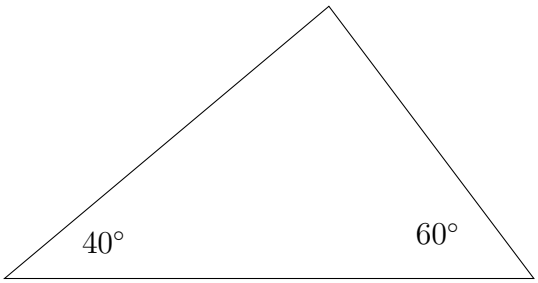
*I think of a number and divide it by 7*

**Common error.** Look at the line $\frac{8}{3} \times 7$. Many people would carry on to write $\frac{56}{21}$ having multiplied the top and bottom by 7. However, remember we are really doing $\frac{8}{3} \times \frac{7}{1}$.

# 2 Similar triangles

We have already seen that similar figures have the same angles. Consider these two triangles:




Since the angles in a triangle add up to $180°$, the missing angle in each triangle is $80°$. Thus, the triangles have equal angles (despite being different sizes) and so must be similar.

| Two triangles are similar if they have two angles the same. |
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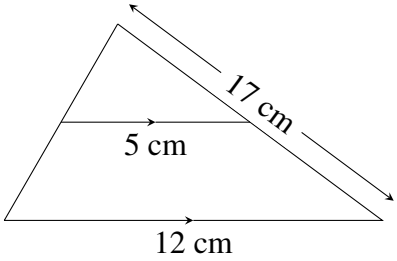
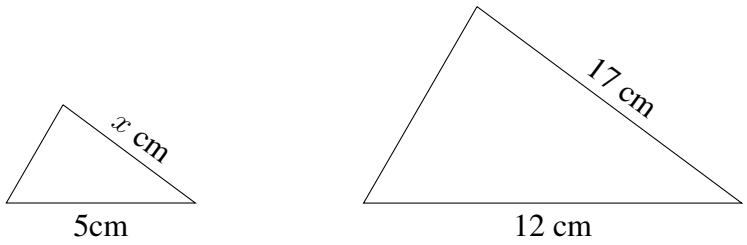
**Example.** Find the length $x$ in the following diagram.




It helps to draw out the two triangles separately first. However, we have not been told they are similar so how do we know they are? The hint comes from the parallel lines – we can easily show that the two angles on the left are equal using corresponding angles. Using the same reasoning, we can see that the two angles on the right are equal as well.
Now we can draw the two triangles separately:




Therefore:

$$\frac{x}{17} = \frac{5}{12}$$
$$x = \frac{5}{12} \times 17$$
$$x = \frac{85}{12}$$
$$x = 7\frac{1}{12} \text{ cm}$$