Trigonometry (9)

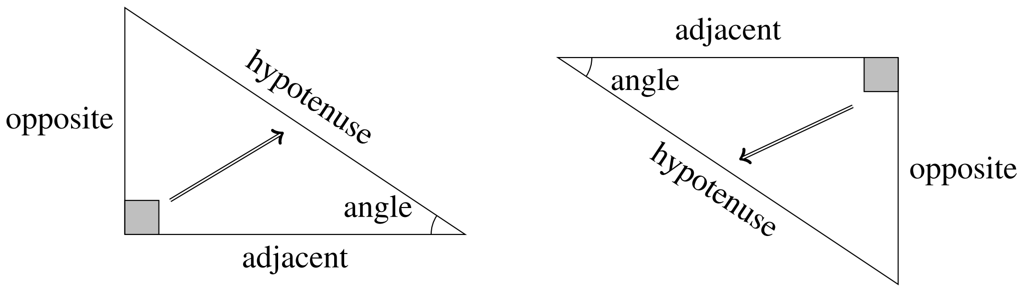
Contents

Introduction

We have already come across Pythagoras' theorem which is used to find missing lengths in right-angled triangles. However, Pythagoras uses two lengths to find a third length. What if we know an angle and a length or wish to find an angle?

1 What is Trigonometry?

Trigonometry is used in right-angled triangles to find a missing length (when we know an angle and a length) or a missing angle (when we know two missing lengths). We need to learn how to label the sides of a triangle — we already know that the hypotenuse is the longest side (opposite the right-angle). The other two sides are labelled according to the angle that we know or want to find:



Always start by labelling the *hypotenuse* and then look opposite the angle that you want or are given to find the *opposite* side. The remaining side is the *adjacent* (this is next to the angle). The three formulae then used are:

$$\text{Sine(angle)} = \frac{\text{Opposite}}{\text{Hypotenuse}} \qquad \text{Cosine(angle)} = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\text{Tangent(angle)} = \frac{\text{Opposite}}{\text{Adjacent}}$$

This can be remembered using **SOHCAHTOA**:

S	Sine
O	Opposite
H	Hypotenuse
C	Cosine
A	Adjacent
H	Hypotenuse
T	Tangent
O	Opposite
A	Adjacent

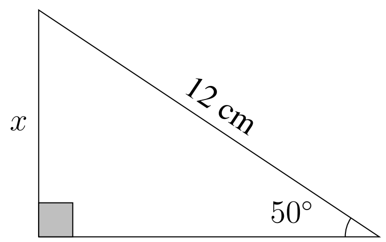
Sine, Cosine and Tangent are abbreviated to $\sin, \cos$ and $\tan$ and will appear as such on your calculator.

N.B. $\sin(\text{angle})$ does not mean "sin $\times$angle", it is one term that cannot be split. E.g. $\sin 300 = 0.5$; it represents a number.
Lets have a look at the three basic uses of trigonometry and then see some applications.

1.1 Finding a side

If you have been given a side and an angle, you can find another side.

Example.



Let's label the sides first:

- the 12 cm side is the hypotenuse
- "x" is the opposite side since it is opposite the angle
- (the remaining side is therefore the adjacent side)

Since we are given the hypotenuse and need to find the opposite, we need to pick the formula involving **O** and **H**, which is the sine formula.

$$\sin(\text{angle}) = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\sin 50° = \frac{x}{12} \qquad \textit{I think of a number, divide it by 12}$$

$$x = 12\sin 50°$$

$$x = 9.192533317\ldots$$

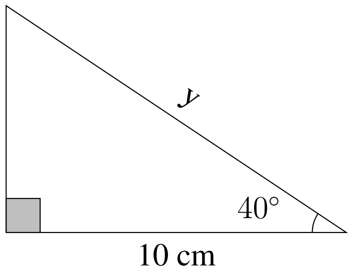
$$x = 9.19 \text{ cm (to 3 s.f.)}$$

this is reasonable since it is less than the hypotenuse which is the longest side

1.2 Finding a side (harder)

The rearrangement is more difficult when the unknown side appears on the bottom of the fraction.

Example.



Let's label the sides first:

- "y" is the hypotenuse
- The blank side is the opposite side since it is opposite the angle
- The 10 cm side is the adjacent side

Since we are given the hypotenuse and need to find the adjacent, we need to pick the formula involving **A** and **H**, which is the cosine formula.

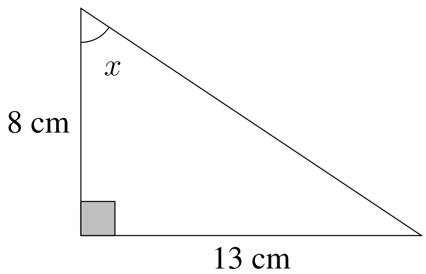
$$\begin{aligned} \cos(\text{angle}) &= \frac{\text{Adjacent}}{\text{Hypotenuse}} \\ \cos 40^\circ &= \frac{10}{y} && \textit{Dividing by } y \textit{ is undone by multiplying by } y \\ y\cos 40^\circ &= 10 \\ y &= \frac{10}{\cos 40^\circ} \\ y &= 13.05407289\ldots \\ y &= 13.05 \text{ cm (to 3 s.f.)} && \textit{this is reasonable since it is less than the hypotenuse which is the longest side} \end{aligned}$$

N.B. Notice that $\cos 40^\circ = \dfrac{10}{y}$ becomes $y = \dfrac{10}{\cos 40^\circ}$. You can imagine that the y and the $\cos 40^\circ$ have simply been swapped.

1.3 Finding an angle

If you have been given two sides, you can find an angle.

Example.



Let's label the sides first:

- The blank side is the hypotenuse
- The 13 cm side is the opposite side since it is opposite the angle
- The 8 cm side is the adjacent side

Since we are given the opposite and the adjacent, we need to pick the formula involving **O** and **A**, which is the tangent formula.

$$\tan(\text{angle}) = \frac{\text{Opposite}}{\text{Adjacent}}$$

$$\tan x = \frac{13}{8}$$

I think of a number and "$\tan$*" it... the opposite is* $\tan^{-1}$

$$x = \tan^{-1}\left(\frac{13}{8}\right)$$

$$x = 58.39249775\ldots$$

$$x = 58.40^\circ \text{ (to 3s.f.)}$$

This is reasonable since the smallest angle is opposite is the smallest side. This is a mid-size length opposite a mid-size angle.

N.B. In all of this work on trigonometry, you must make sure your calculator is in *degree* mode: ask your teacher if unsure.

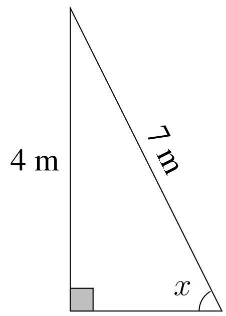
Have you learned these 3 trig rearrangements?

2 Applied Questions

You can use your trig skills in a variety of questions. Study the examples below.

Example. A ladder of length 7 m leans against a vertical wall. The foot of the ladder is 4 m from the wall. What angle does the ladder make with the floor?

A diagram is always useful:



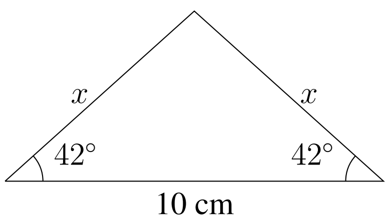
$$\sin(\text{angle}) = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$\sin x = \frac{4}{7}$$

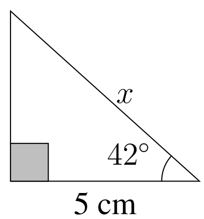
$$x = 34.84990458\ldots$$

$$x = 34.8^\circ \text{ (to 3 s.f.)}$$

Example. Find the length of the equal sides of an isosceles triangle with base10 cm and equal angles 42°?



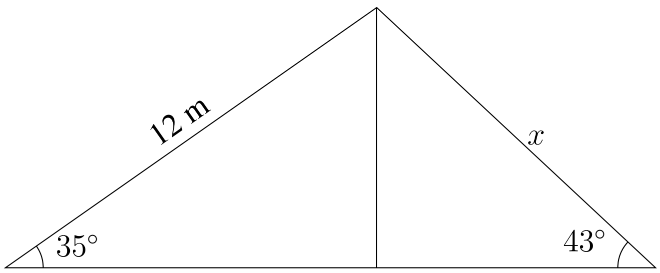
We can split this triangle in two along the vertical height:



Looking at the right-angle triangle:

$$\cos(\text{angle}) = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$
$$\cos 42^\circ = \frac{5}{x}$$
$$x = \frac{5}{\cos 42^\circ}$$
$$x = 6.7281636\ldots$$
$$x = 6.73 \text{ cm (to 3 s.f.)}$$

Example. Find the missing length in the following diagram:



We need to find a missing length in the triangle on the right, but we only have one piece of information in this triangle (we need two to use the trig formulae). However, we can use the left triangle to find the length that the two triangles share in the middle (lets call this y) and then use this to find x:

$$\sin(\text{angle}) = \frac{\text{Opposite}}{\text{Hypotenuse}}$$ *Find the vertical side in the left triangle*

$$\sin 35^\circ = \frac{y}{12}$$
$$y = 12\sin 35^\circ$$ *There is no need to work this out as a decimal as it is not the final answer*

$$\sin(\text{angle}) = \frac{\text{Opposite}}{\text{Hypotenuse}}$$ *Find the horizontal side in the right triangle*

$$\sin 43^\circ = \frac{y}{x}$$
$$\sin 43^\circ = \frac{12\sin 35^\circ}{x}$$
$$x = \frac{12\sin 35^\circ}{\sin 43^\circ}$$
$$x = 9.41120175\ldots$$
$$x = 9.41 \text{ m (to 3 s.f.)}$$

Don’t forget to learn your trig work and formulae – this is a level 8 topic and a useful source of “easy” marks when it arises in the SATs